12 Turbomachines

CHAPTER OPENING PHOTO: A mixed-flow, transonic compressor stage. (Photograph courtesy of Concepts NREC.)

Learning Objectives

After completing this chapter, you should be able to:

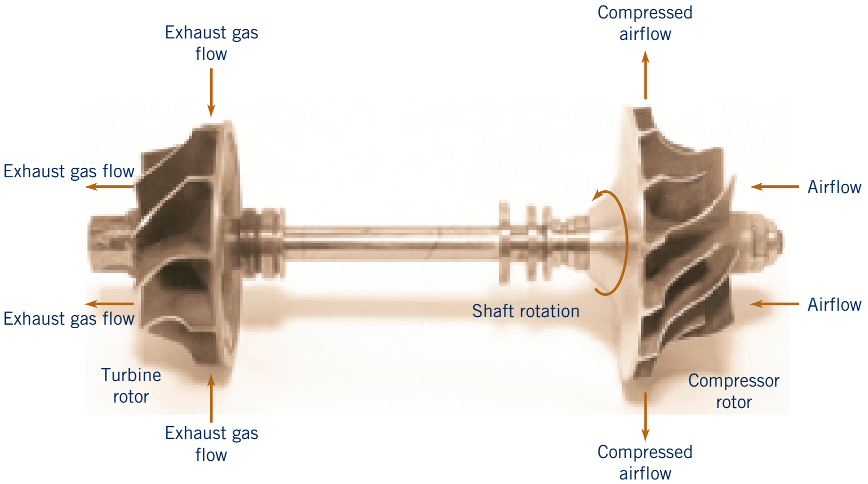
- explain how and why a turbomachine works.
- know the basic differences between a turbine and a pump.
- recognize the importance of minimizing loss in a turbomachine.
- select an appropriate class of turbomachines for a particular application.
- understand why turbomachine blades are shaped like they are.
- appreciate the basic fundamentals of sensibly scaling turbomachines that are larger or smaller than a prototype.
- move on to more advanced engineering work involving the fluid mechanics of turbomachinery (e.g., design, development, research).

In previous chapters we often used generic "black boxes" to represent fluid machines such as pumps or turbines. The purpose of this chapter is to understand the fluid mechanics of these devices when they are turbomachines.

Pumps and turbines (often turbomachines) occur in a wide variety of configurations. In general, pumps add energy to the fluid—they do work on the fluid to move and/or increase the pressure of the fluid; turbines extract energy from the fluid—the fluid does work on them. The term "pump" will be used to generically refer to all pumping machines, including *pumps*, *fans*, *blowers*, and *compressors*.

Turbomachines are dynamic fluid machines that add (for pumps) or extract (for turbines) flow energy.

Turbomachines involve a collection of blades, buckets, flow channels, or passages arranged around an axis of rotation to form a rotor. A fluid that is moving can force rotation and produce shaft power. In this case we have a turbine. On the other hand, we can exert a shaft torque, typically with a motor, and by using blades, flow channels, or passages force the fluid to move. In this case we have a pump. In Fig. 12.1 are shown the turbine and compressor (pump) rotors of an automobile turbocharger. Examples of turbomachine-type pumps include simple window fans, propellers on ships or airplanes, squirrel-cage fans on home furnaces, axial-flow water



■ **Figure 12.1** **Automotive turbocharger turbine and compressor rotors. (Photograph courtesy of Concepts NREC.)**

pumps used in deep wells, and compressors in automobile turbochargers. Examples of turbines include the turbine portion of gas turbine engines on aircraft, steam turbines used to drive generators at electrical generation stations, and the small, high-speed air turbines that power dentist drills.

Turbomachines serve in an enormous array of applications in our daily lives and thus play an important role in modern society. These machines can have a high-power density (large-power transfer per size), relatively few moving parts, and reasonable efficiency. The following sections provide an introduction to the fluid mechanics of these important machines. References 1–3 are a few examples of the many books that offer much more knowledge about turbomachines.

12.1 Introduction

Turbomachines involve the related parameters of force, torque, work, and power.

(Photograph courtesy of MidAmerican Energy.)

Turbomachines are mechanical devices that either extract energy from a fluid (turbine) or add energy to a fluid (pump) as a result of dynamic interactions between the device and the fluid. While the actual design and construction of these devices often require considerable insight and effort, their basic operating principles are quite simple.

Using a food blender to make a fruit drink is an example of turbo-pump action. The blender blades are forced to rotate around an axis by a motor. The moving blades pulverize fruit and ice and mix them with a base liquid to form a "smoothie."

Conversely, the dynamic effect of the wind blowing past the sail on a boat creates pressure differences on the sail. The wind force on the moving sail in the direction of the boat's motion provides power to propel the boat. The sail and boat act as a machine extracting energy from the air. Turbine blades are like sails. See, for example, the enormous wind turbine blades in the figure in the margin.

The fluid involved can be either a gas (as with a window fan or a gas turbine engine) or a liquid (as with the water pump on a car or a turbine at a hydroelectric power plant). While the basic operating principles are the same whether the fluid is a liquid or a gas, important differences in the fluid dynamics involved can occur. For example, cavitation may be an important design consideration when liquids are involved if the pressure at any point within the flow is reduced to the vapor pressure. Compressibility effects may be important when gases are involved if the Mach number becomes large enough.

Many turbomachines contain some type of housing or casing that surrounds the rotating blades or rotor, thus forming an internal flow passageway through which the fluid flows (see Fig. 12.2). Others, such as a windmill or a window fan, are unducted. Some turbomachines

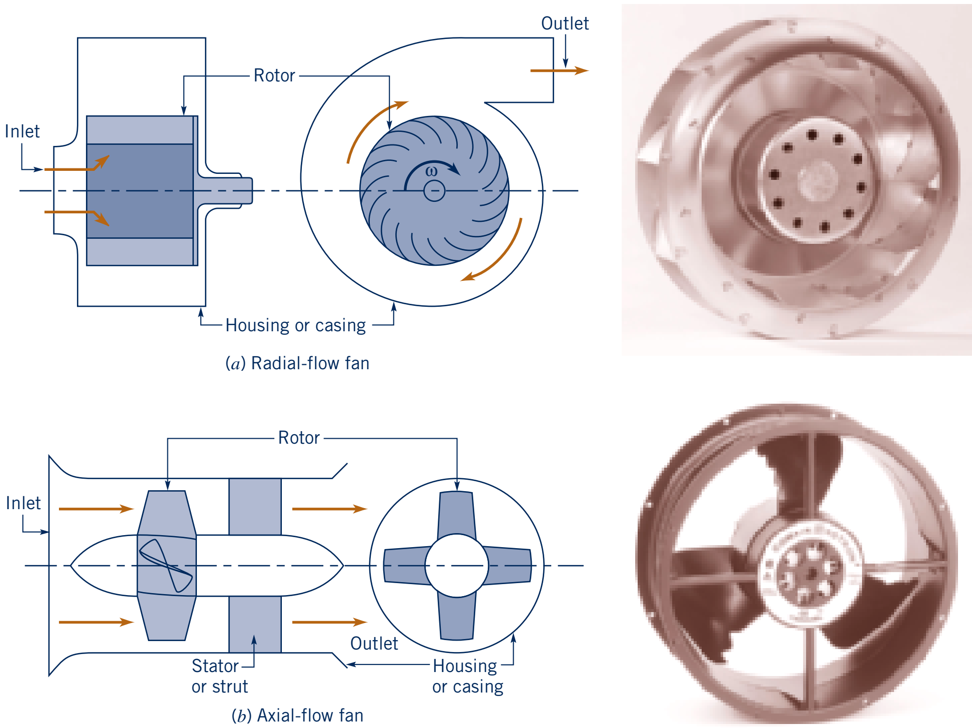


Figure 12.2 **(*a*) A radial-flow turbomachine, (*b*) an axial-flow turbomachine. (Photographs courtesy of Comair Rotron, Inc.)**

A group of blades moving with or against a lift force is the essence of a turbomachine.

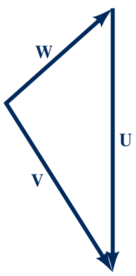
include stationary blades or vanes in addition to rotor blades. These stationary vanes can be arranged to accelerate the flow and thus serve as nozzles. Or these vanes can be set to diffuse the flow and act as diffusers.

Turbomachines are classified as ***axial-flow***, ***mixed-flow***, or ***radial-flow*** machines depending on the predominant direction of the fluid motion relative to the rotor's axis as the fluid passes the blades (see Fig. 12.2). For an axial-flow machine the fluid maintains a significant axial-flow direction component from the inlet to outlet of the rotor. For a radial-flow machine the flow across the blades involves a substantial radial-flow component at the rotor inlet, exit, or both. In other machines, designated as mixed-flow machines, there may be significant radial- and axial-flow velocity components for the flow through the rotor row. Each type of machine has advantages and disadvantages for different applications and in terms of fluid-mechanical performance.

12.2 Basic Energy Considerations

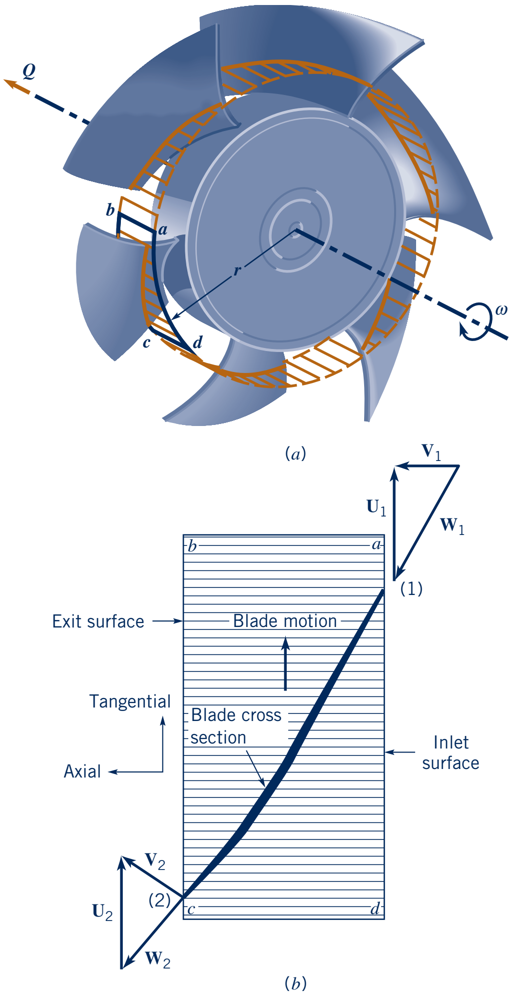
An understanding of the work transfer in turbomachines can be obtained by considering the basic operation of a household fan (pump) and a windmill (turbine). Although the actual flows in such devices are very complex (i.e., three-dimensional and unsteady), the essential phenomena can be illustrated by use of simplified flow considerations and velocity triangles.

Consider a fan blade driven at constant angular velocity, ω, by a motor as is shown in Fig. 12.3*a*. We denote the blade speed as $U = \omega r$, where r is the radial distance from the axis of the fan. The absolute fluid velocity (that seen by a person sitting stationary at the table on which the fan rests) is denoted $\mathbf{V}$, and the relative velocity (that seen by a person riding on the fan blade) is denoted $\mathbf{W}$. As shown by the figure in the margin, the actual (absolute) fluid velocity is the vector sum of the relative velocity and the blade velocity



$$\mathbf{V} = \mathbf{W} + \mathbf{U} \tag{12.1}$$

A simplified sketch of the fluid velocity as it "enters" and "exits" the fan at radius r is shown in Fig. 12.3*b*. The shaded surface labeled a–b–c–d is a portion of the cylindrical surface (including a "slice" through the blade) shown in Fig. 12.3*a*. We assume for simplicity that the



■ **Figure 12.3** **Idealized flow through a fan: (*a*) fan blade geometry; (*b*) absolute velocity, V; relative velocity, W; and blade velocity, U at the inlet and exit of the fan blade section.**

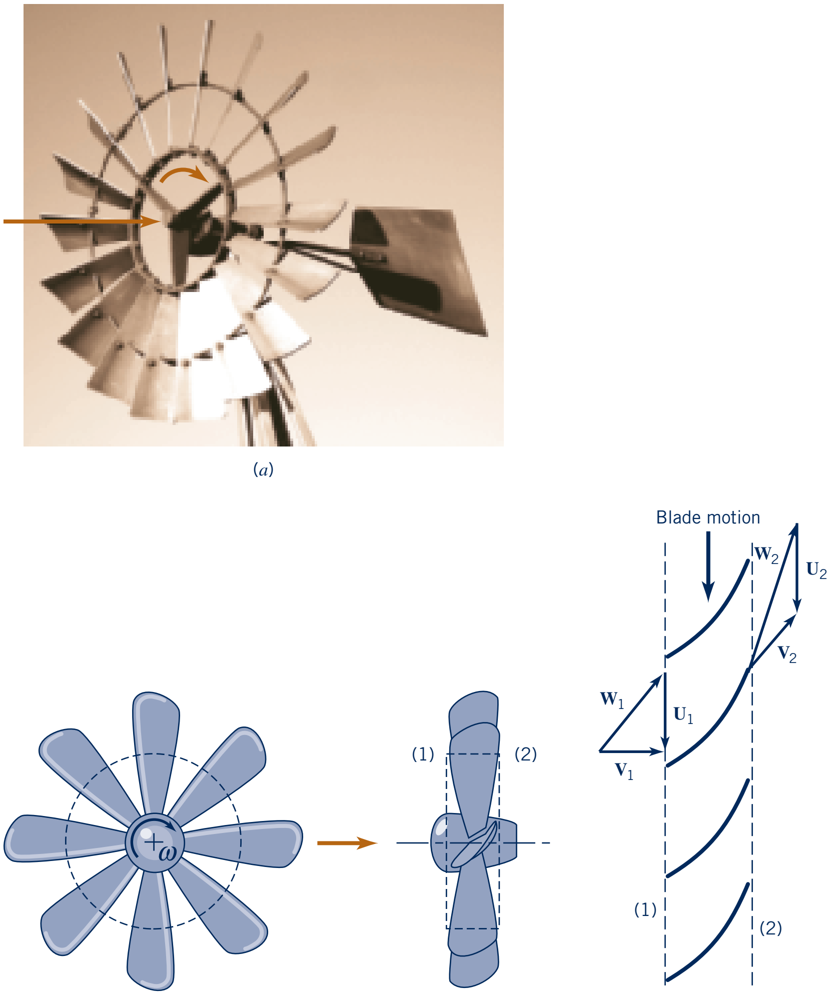
flow moves smoothly along the blade so that relative to the moving blade the velocity is parallel to the leading and trailing edges (points 1 and 2) of the blade. For now we assume that the fluid enters and leaves the fan at the same distance from the axis of rotation; thus, $U_1 = U_2 = \omega r$. In actual turbomachines, the entering and leaving flows are not necessarily tangent to the blades, and the fluid pathlines can involve changes in radius. These considerations are important at design and off-design operating conditions. Interested readers are referred to Refs. 1, 2, and 3 for more information about these aspects of turbomachine flows.

With this information we can construct the ***velocity triangles*** shown in Fig. 12.3*b*. Note that this view is from the top of the fan, looking radially down toward the axis of rotation. The motion of the blade is up; the motion of the incoming air is assumed to be directed along the axis of rotation. The important concept to grasp from this sketch is that the fan blade (because of its shape and motion) "pushes" the fluid, causing it to change direction. The absolute velocity vector, **V**, is turned during its flow across the blade from section (1) to section (2). Initially the fluid had no component of absolute velocity in the direction of the motion of the blade, the θ (or tangential) direction. When the fluid leaves the blade, this tangential component of absolute velocity is nonzero. For this to occur, the blade must push on the fluid in the tangential direction. That is, the blade exerts a tangential force component on the fluid in the direction of the motion of the blade. This tangential force component and the blade motion are in the same direction—the blade does work on the fluid. This device is a pump.

When blades move because of the fluid force, we have a turbine; when blades are forced to move fluid, we have a pump.

On the other hand, consider the windmill shown in Fig. 12.4*a* (also see **Video V12.1**). Rather than the rotor being driven by a motor, the blades move in the direction of the lift force (compared to the fan in Fig. 12.3) exerted on each blade by the wind blowing through the rotor. We again note that because of the blade shape and motion, the absolute velocity vectors at sections (1) and (2), $\mathbf{V}_1$

V12.1 Windmills



(b) (c)

■ **Figure 12.4 Idealized flow through a windmill: (*a*) windmill; (*b*) windmill blade geometry; (*c*) absolute velocity, V; relative velocity, W; and blade velocity, U; at the inlet and exit of the windmill blade section.**

and $\mathbf{V}_2$, have different directions. For this to happen, the blades must have pushed up on the fluid—opposite to the direction of blade motion. Alternatively, because of equal and opposite forces (action/reaction) the fluid must have pushed on the blades in the direction of their motion—the fluid does work on the blades. This extraction of energy from the fluid is the purpose of a turbine.

These examples involve work transfer to or from a flowing fluid in two axial-flow turbomachines. Similar concepts hold for other turbomachines including mixed-flow and radial-flow configurations.

Fluids in the News

Current from currents The use of large, efficient wind *turbines* to generate electrical power is becoming more commonplace throughout the world. "Wind farms" containing numerous turbines located at sites that have proper wind conditions can produce a significant amount of electrical power. Recently, researchers in the United States, the United Kingdom, and Canada have been investigating the possibility of harvesting the power of ocean currents and tides by using current turbines that function much like wind turbines. Rather than being driven by wind, they derive energy from ocean currents that occur at many locations in the 70% of the Earth's surface that is water. Clearly, a 2.5 m/s tidal current is not as fast as a 70 km/h wind driving a wind turbine. However, since turbine power output is proportional to the fluid density, and since seawater is more than 800 times as dense as air, significant power can be extracted from slow, but massive, ocean currents. One promising configuration involves *blades* twisted in a helical pattern. This technology may provide electrical power that is both ecologically and economically sound.

EXAMPLE 12.1 Basic Difference between a Pump and a Turbine

GIVEN The rotor shown in Fig. E12.1*a* rotates at a constant angular velocity of $\omega = 100$ rad/s. Although the fluid initially approaches the rotor in an axial direction, the flow across the blades is primarily outward (see Fig. 12.2*a*). Measurements indicate that the absolute velocity at the inlet and outlet are $V_1 = 12$ m/s and $V_2 = 15$ m/s, respectively.

FIND Is this device a pump or a turbine?

SOLUTION

To answer this question, we need to know if the tangential component of the force of the blade on the fluid is in the direction of the blade motion (a pump) or opposite to it (a turbine). We assume that the blades are tangent to the incoming relative velocity and that the relative flow leaving the rotor is tangent to the blades as shown in Fig. E12.1*b*. We can also calculate the inlet and outlet blade speeds as

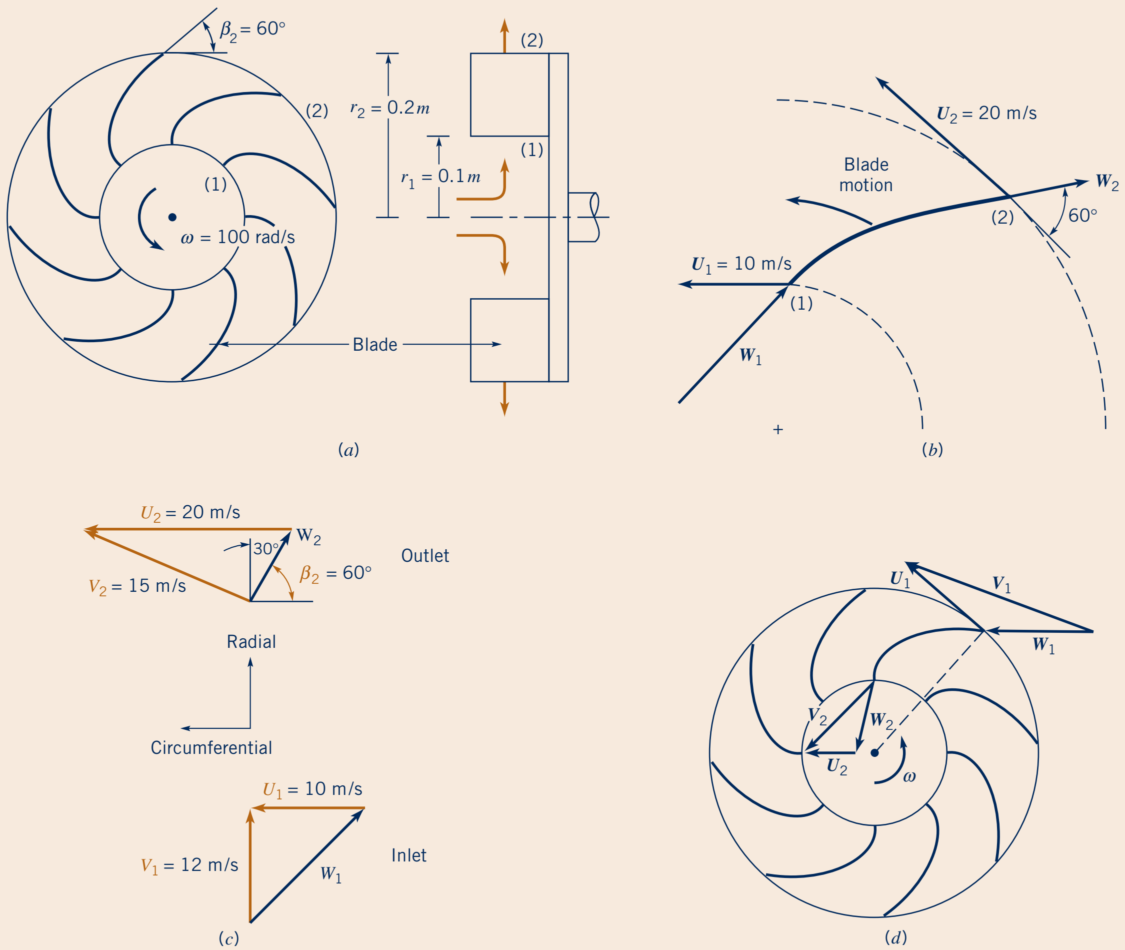
$$U_1 = \omega r_1 = (100 \text{ rad/s})(0.1 \text{ m}) = 10 \text{ m/s}$$

and

$$U_2 = \omega r_2 = (100 \text{ rad/s})(0.2 \text{ m}) = 20 \text{ m/s}$$

With the known, absolute fluid velocity and blade velocity at the inlet, we can draw the velocity triangle (the graphical representation of Eq. 12.1) at that location as shown in Fig. E12.1*c*. Note that we have assumed that the absolute flow at the blade row inlet is radial (i.e., the direction of $\mathbf{V}_1$ is radial). At the outlet we know the blade velocity, $\mathbf{U}_2$, the outlet speed, V_2, and the relative velocity direction, β_2 (because of the blade geometry). Therefore, we can graphically (or trigonometrically) construct the outlet velocity triangle as shown in the figure. By comparing the velocity triangles at the inlet and outlet, it can be seen that as the fluid flows across the blade row, the absolute velocity vector turns in the direction of the blade motion. At the inlet there is no component of absolute velocity in the direction of rotation; at the outlet this component is not zero. That is, the blade pushes and turns the fluid in the direction of the blade motion, thereby doing work on the fluid, adding energy to it.

This device is a pump. **(Ans)**



■ **Figure E12.1**

COMMENT On the other hand, by reversing the direction of flow from larger to smaller radii, this device can become a radial-flow turbine. In this case (Fig. E12.1*d*) the flow direction is reversed (compared to that in Figs. E12.1*a*, *b*, and *c*) and the velocity triangles are as indicated. Stationary vanes around the perimeter of the rotor would be needed to achieve $\mathbf{V}_1$ as shown. Note that the component of the absolute velocity, $\mathbf{V}$, in the direction of the blade motion is smaller at the outlet than at the inlet. The blade must push against the fluid in the direction opposite the motion of the blade to cause this. Hence (by equal and opposite forces), the fluid pushes against the blade in the direction of blade motion, thereby doing work on the blade. There is a transfer of work from the fluid to the blade—a turbine operation.

12.3 Basic Angular Momentum Considerations

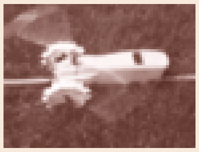
In the previous section we indicated how work transfer to or from a fluid flowing through a pump or a turbine occurs by interaction between moving rotor blades and the fluid. Since all of these turbomachines involve the rotation of an impeller or a rotor about a central axis, it is appropriate to discuss their performance in terms of torque and angular momentum.

Recall that work can be written as force times distance or as torque times angular displacement. Hence, if the shaft torque (the torque that the shaft applies to the rotor) and the rotation of the rotor are in the same direction, energy is transferred from the shaft to the rotor and from the rotor to the fluid—the machine is a pump. Conversely, if the torque exerted by the shaft on the rotor is opposite to the direction of rotation, the energy transfer is from the fluid to the rotor—a turbine. The amount of shaft torque (and, hence, shaft work) can be obtained from the moment-of-momentum equation derived formally in Section 5.2.3 and discussed as follows.

When shaft torque and rotation are in the same direction, we have a pump; otherwise we have a turbine.

Consider a fluid particle traveling outward through the rotor in the radial-flow machine shown in Figs. E12.1*a*, *b*, and *c*. For now, assume that the particle enters the rotor with a radial velocity only (i.e., no "swirl"). After being acted upon by the rotor blades during its passage from the inlet [section (1)] to the outlet [section (2)], this particle exits with radial (r) and circumferential (θ) components of velocity. Thus, the particle enters with no angular momentum about the rotor axis of rotation but leaves with nonzero angular momentum about that axis. (Recall that the axial component of angular momentum for a particle is its mass times the distance from the axis times the θ component of absolute velocity.)

V12.2 Self-propelled lawn sprinkler



A similar experience can occur at the neighborhood playground. Consider yourself as a particle and a merry-go-round as a rotor. Walk from the center to the edge of the spinning merry-go-round and note the forces involved. The merry-go-round does work on you—there is a "sideward force" on you. Another person must apply a torque (and power) to the merry-go-round to maintain a constant angular velocity; otherwise the angular momentum of the system (you and the merry-go-round) is conserved, and the angular velocity decreases as you increase your distance from the axis of rotation. (Similarly, if the motor driving a pump is turned off, the pump will obviously slow down and stop.) Your friend is the motor supplying energy to the rotor that is transferred to you. Is the amount of energy your friend expends to keep the angular velocity constant dependent on what path you follow along the merry-go-round (i.e., the blade shape); on how fast and in what direction you walk off the edge (i.e., the exit velocity); on how much you weigh (i.e., the density of the fluid)? What happens if you walk from the outside edge toward the center of the rotating merry-go-round? Recall that the opposite of a pump is a turbine.

In a turbomachine a series of particles (a continuum) passes through the rotor. Thus, the moment-of-momentum equation applied to a control volume as derived in Section 5.2.3 is valid. For steady flow (or for turbomachine rotors with steady-in-the-mean or steady-on-average cyclical flow), Eq. 5.42 gives

$$\sum (\mathbf{r} \times \mathbf{F}) = \int_{cs} (\mathbf{r} \times \mathbf{V})\, \rho \mathbf{V} \cdot \hat{\mathbf{n}}\, dA$$

Recall that the left-hand side of this equation represents the sum of the external torques (moments) acting on the contents of the control volume, and the right-hand side is the net rate of flow of moment-of-momentum (***angular momentum***) through the control surface.

The axial component of this equation applied to the one-dimensional simplification of flow through a turbomachine rotor with section (1) as the inlet and section (2) as the outlet results in

$$T_{\text{shaft}} = -\dot{m}_1(r_1 V_{\theta 1}) + \dot{m}_2(r_2 V_{\theta 2}) \tag{12.2}$$

where T_{shaft} is the ***shaft torque*** applied to the contents of the control volume. The "$-$" is associated with mass flowrate into the control volume, and the " $+$ " is used with the outflow. The sign of the V_θ component depends on the direction of V_θ and the blade motion, U. If V_θ and U are in the same direction, then V_θ is positive. The sign of the torque exerted by the shaft on the rotor, T_{shaft}, is positive if T_{shaft} is in the same direction as rotation, and negative otherwise.

As seen from Eq. 12.2, the shaft torque is directly proportional to the mass flowrate, $\dot{m} = \rho Q$. (It takes considerably more torque and power to pump water than to pump air with the same volume flowrate.) The torque also depends on the tangential component of the absolute velocity, V_θ. Equation 12.2 is often called the ***Euler turbomachine equation***.

The Euler turbomachine equation is the axial component of the moment-of-momentum equation.

Also recall that the ***shaft power***, $\dot{W}_{\text{shaft}}$, is related to the shaft torque and angular velocity by

$$\dot{W}_{\text{shaft}} = T_{\text{shaft}}\, \omega \tag{12.3}$$

By combining Eqs. 12.2 and 12.3 and using the fact that $U = \omega r$, we obtain

$$\dot{W}_{\text{shaft}} = -\dot{m}_1(U_1 V_{\theta 1}) + \dot{m}_2(U_2 V_{\theta 2}) \tag{12.4}$$

Again, the value of V_θ is positive when V_θ and U are in the same direction and negative otherwise. Also, $\dot{W}_{\text{shaft}}$ is positive when the shaft torque and ω are in the same direction and negative otherwise. Thus, $\dot{W}_{\text{shaft}}$ is positive when power is supplied to the contents of the control volume (pumps) and negative otherwise (turbines). This outcome is consistent with the sign convention involving the work term in the energy equation considered in Chapter 5 (see Eq. 5.67).

Finally, in terms of work per unit mass, $w_{\text{shaft}} = \dot{W}_{\text{shaft}}/\dot{m}$, we obtain

$$w_{\text{shaft}} = -U_1 V_{\theta 1} + U_2 V_{\theta 2} \tag{12.5}$$

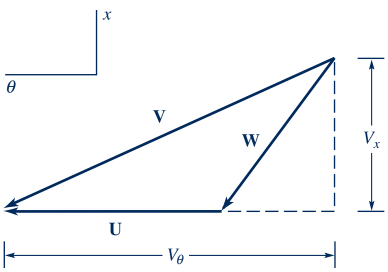
where we have used the fact that by conservation of mass, $\dot{m}_1 = \dot{m}_2$. Equations 12.3, 12.4, and 12.5 are the basic governing equations for pumps or turbines whether the machines are radial-, mixed-, or axial-flow devices and for compressible and incompressible flows. Note that neither the axial nor the radial component of velocity enter into the specific work (work per unit mass) equation. [In the above merry-go-round example the amount of work your friend does is independent of how fast you jump "up" (axially) or "out" (radially) as you exit. The only thing that counts is your θ component of velocity.]

Another useful but more laborious form of Eq. 12.5 can be obtained by writing the right-hand side in a slightly different form based on the velocity triangles at the entrance or exit as shown generically in Fig. 12.5. The velocity component V_x is the generic through-flow component of velocity and it can be axial, radial, or in-between depending on the rotor configuration. From the large right triangle we note that

$$V^2 = V_\theta^2 + V_x^2$$

or

$$V_x^2 = V^2 - V_\theta^2 \tag{12.6}$$



■ **Figure 12.5** **Velocity triangle: V = absolute velocity, W = relative velocity, U = blade velocity.**

From the small right triangle we note that

$$V_x^2 + (V_\theta - U)^2 = W^2 \tag{12.7}$$

By combining Eqs. 12.6 and 12.7 we obtain

$$V_\theta U = \frac{V^2 + U^2 - W^2}{2}$$

which when written for the inlet and exit and combined with Eq. 12.5 gives

$$w_{\text{shaft}} = \frac{V_2^2 - V_1^2 + U_2^2 - U_1^2 - (W_2^2 - W_1^2)}{2} \tag{12.8}$$

Turbomachine work is related to changes in absolute, relative, and blade velocities.

Thus, the power and the shaft work per unit mass can be obtained from the speed of the blade, U, the absolute fluid speed, V, and the fluid speed relative to the blade, W. This is an alternative to using fewer components of the velocity as suggested by Eq. 12.5. Equation 12.8 contains more terms than Eq. 12.5; however, it is an important concept equation because it shows how the work transfer is related to absolute, relative, and blade velocity changes. Because of the general nature of the velocity triangle in Fig. 12.5, Eq. 12.8 is applicable for axial-, radial-, and mixed-flow rotors.

Fluids in the News

1948 Buick Dynaflow started it Prior to 1948 almost all cars had manual transmissions, which required the use of a clutch pedal to shift gears. The 1948 Buick Dynaflow was the first automatic transmission to use the hydraulic *torque* converter and was the model for present-day automatic transmissions. Currently, in the United States over 84% of the cars have automatic transmissions. The torque converter replaces the clutch found on manual shift vehicles and allows the engine to continue running when the vehicle comes to a stop. In principle, but certainly not in detail or complexity, operation of a torque converter is similar to blowing air from a *fan* onto another fan that is unplugged. One can hold the *blade* of the unplugged fan and keep it from turning, but as soon as it is let go, it will begin to speed up until it comes close to the speed of the powered fan. The torque converter uses transmission *fluid* (not air) and consists of a *pump* (the powered fan) driven by the engine driveshaft, a *turbine* (the unplugged fan) connected to the input shaft of the transmission, and a *stator* (absent in the fan model) to efficiently direct the flow between the pump and turbine.

12.4 The Centrifugal Pump

One of the most common radial-flow turbomachines is the ***centrifugal pump***. This type of pump has two main components: an *impeller* attached to a rotating shaft, and a stationary *casing*, *housing*, or *volute* enclosing the impeller. The impeller consists of a number of blades (usually curved), also sometimes called *vanes*, arranged in a regular pattern around the shaft. A sketch showing the essential features of a centrifugal pump is shown in Fig. 12.6. As the impeller rotates, fluid is sucked in through the *eye* of the casing and flows radially outward. Energy is added to the fluid by the rotating blades, and both pressure and absolute velocity are increased as the fluid flows from the eye to the periphery of the blades. For the simplest type of centrifugal pump, the

V12.3 Windshield washer pump

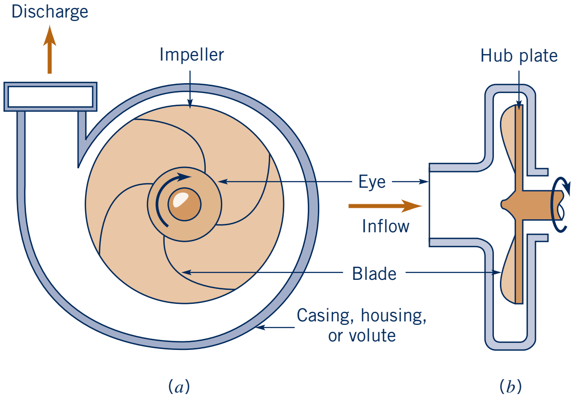


Figure 12.6 Schematic diagram of basic elements of a centrifugal pump.

(*a*)

(*b*)

■ **Figure 12.7** **(*a*) Open impeller, (*b*) enclosed or shrouded impeller. (Courtesy of Ingersoll-Dresser Pump Company.)**

fluid discharges directly into a volute-shaped casing. The casing shape is designed to reduce the velocity as the fluid leaves the impeller, and this decrease in kinetic energy is converted into an increase in pressure. The volute-shaped casing, with its increasing area in the direction of flow, is used to produce an essentially uniform velocity distribution as the fluid moves around the casing into the discharge opening. For large centrifugal pumps, a different design is often used in which diffuser guide vanes surround the impeller. The diffuser vanes decelerate the flow as the fluid is directed into the pump casing. This type of centrifugal pump is referred to as a *diffuser* pump.

Impellers are generally of two types. For one configuration the blades are arranged on a hub or backing plate and are open on the other (casing or shroud) side. A typical *open impeller* is shown in Fig. 12.7*a*. For the second type of impeller, called an *enclosed* or *shrouded* impeller, the blades are covered on both hub and shroud ends as shown in Fig. 12.7*b*.

Pump impellers can also be *single* or *double suction*. For the single-suction impeller the fluid enters through the eye on only one side of the impeller, whereas for the double-suction impeller the fluid enters the impeller along its axis from both sides. The double-suction arrangement reduces end thrust on the shaft, and also, since the net inlet flow area is larger, inlet velocities are reduced.

Pumps can be *single* or *multistage*. For a single-stage pump, only one impeller is mounted on the shaft, whereas for multistage pumps, several impellers are mounted on the same shaft. The stages operate in series; that is, the discharge from the first stage flows into the eye of the second stage, the discharge from the second stage flows into the eye of the third stage, and so on. The flowrate is the same through all stages, but each stage develops an additional pressure rise. Thus, a very large discharge pressure, or head, can be developed by a multistage pump.

Centrifugal pumps involve radially outward flows.

Centrifugal pumps come in a variety of arrangements (open or shrouded impellers, volute or diffuser casings, single- or double-suction, single- or multistage), but the basic operating principle remains the same. Work is done on the fluid by the rotating blades (centrifugal action and tangential blade force acting on the fluid over a distance), creating a large increase in kinetic energy of the fluid flowing through the impeller. This kinetic energy is converted into an increase in pressure as the fluid flows from the impeller into the casing enclosing the impeller. A simplified theory describing the behavior of the centrifugal pump was introduced in the previous section and is expanded in the following section.

12.4.1 Theoretical Considerations

Although flow through a pump is very complex (unsteady and three-dimensional), the basic theory of operation of a centrifugal pump can be developed by considering the average one-dimensional flow of the fluid as it passes between the inlet and the outlet sections of the impeller as the blades rotate. As shown in Fig. 12.8, for a typical blade passage, the absolute velocity, $\mathbf{V}_1$, of the fluid entering the passage is the vector sum of the velocity of the blade, $\mathbf{U}_1$, rotating in a circular path with angular velocity ω, and the relative velocity, $\mathbf{W}_1$, within the blade passage so that $\mathbf{V}_1 = \mathbf{W}_1 + \mathbf{U}_1$. Similarly, at the exit $\mathbf{V}_2 = \mathbf{W}_2 + \mathbf{U}_2$. Note that $U_1 = r_1\omega$ and $U_2 = r_2\omega$. Fluid velocities are taken to be average velocities over the inlet and exit sections of the blade passage. The relationship between the various velocities is shown graphically in Fig. 12.8.

As discussed in Section 12.3, the moment-of-momentum equation indicates that the shaft torque, T_{shaft}, required to rotate the pump impeller is given by equation Eq. 12.2 applied to a pump with $\dot{m}_1 = \dot{m}_2 = \dot{m}$. That is,

$$T_{\text{shaft}} = \dot{m}(r_2 V_{\theta 2} - r_1 V_{\theta 1}) \quad \textbf{(12.9)}$$

or

$$T_{\text{shaft}} = \rho Q(r_2 V_{\theta 2} - r_1 V_{\theta 1}) \quad \textbf{(12.10)}$$

where $V_{\theta 1}$ and $V_{\theta 2}$ are the tangential components of the absolute velocities, $\mathbf{V}_1$ and $\mathbf{V}_2$ (see Figs. 12.8*b*,*c*).

For a rotating shaft, the power transferred, $\dot{W}_{\text{shaft}}$, is given by

$$\dot{W}_{\text{shaft}} = T_{\text{shaft}}\omega$$

and, therefore, from Eq. 12.10

$$\dot{W}_{\text{shaft}} = \rho Q \omega (r_2 V_{\theta 2} - r_1 V_{\theta 1})$$

Centrifugal pump impellers involve an increase in blade velocity along the flow path.

Since $U_1 = r_1\omega$ and $U_2 = r_2\omega$ we obtain

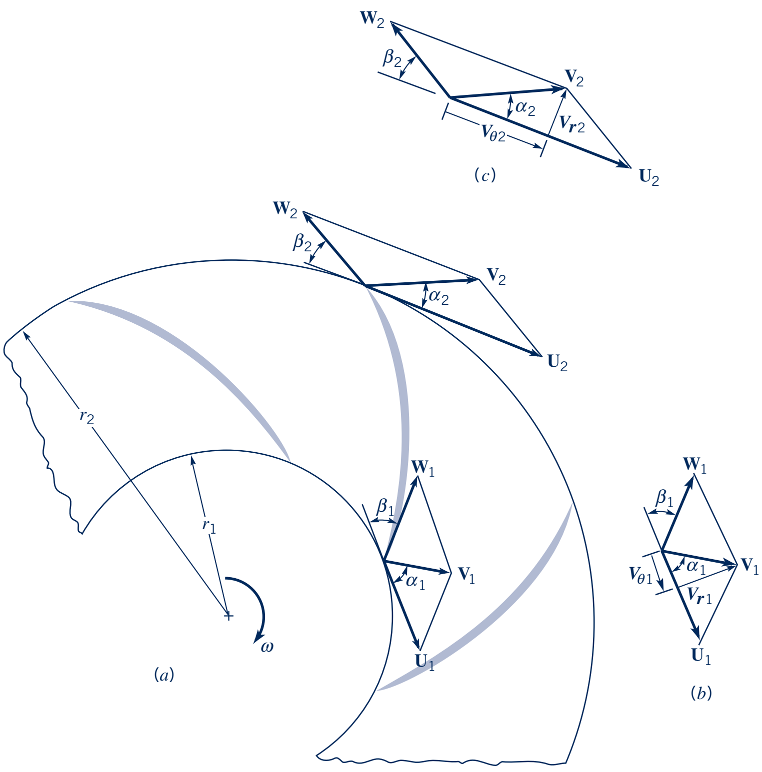
$$\dot{W}_{\text{shaft}} = \rho Q(U_2 V_{\theta 2} - U_1 V_{\theta 1}) \quad \textbf{(12.11)}$$

Equation 12.11 shows how the power supplied to the shaft of the pump is transferred to the flowing fluid. It also follows that the shaft power per unit mass of flowing fluid is

$$w_{\text{shaft}} = \frac{\dot{W}_{\text{shaft}}}{\rho Q} = U_2 V_{\theta 2} - U_1 V_{\theta 1} \quad \textbf{(12.12)}$$

For incompressible pump flow, we get from Eq. 5.82

$$w_{\text{shaft}} = \left(\frac{p_{\text{out}}}{\rho} + \frac{V_{\text{out}}^2}{2} + gz_{\text{out}}\right) - \left(\frac{p_{\text{in}}}{\rho} + \frac{V_{\text{in}}^2}{2} + gz_{\text{in}}\right) + loss$$



■ **Figure 12.8** **Velocity diagrams at the inlet and exit of a centrifugal pump impeller.**

Combining Eq. 12.12 with this, we get

$$U_2V_{\theta 2} - U_1V_{\theta 1} = \left(\frac{p_{\text{out}}}{\rho} + \frac{V_{\text{out}}^2}{2} + gz_{\text{out}}\right) - \left(\frac{p_{\text{in}}}{\rho} + \frac{V_{\text{in}}^2}{2} + gz_{\text{in}}\right) + loss$$

Dividing both sides of this equation by the acceleration of gravity, g, we obtain

$$\frac{U_2V_{\theta 2} - U_1V_{\theta 1}}{g} = H_{\text{out}} - H_{\text{in}} + h_L$$

where H is total head defined by

$$H = \frac{p}{\rho g} + \frac{V^2}{2g} + z$$

and h_L = loss/g is head loss.

From this equation we see that $(U_2V_{\theta 2} - U_1V_{\theta 1})/g$ is the shaft work head added to the fluid by the pump. Head loss, h_L, reduces the actual head rise, $H_{\text{out}} - H_{\text{in}}$, achieved by the fluid. Thus, the ideal head rise possible, h_i, is

$$h_i = \frac{U_2V_{\theta 2} - U_1V_{\theta 1}}{g} \quad \textbf{(12.13)}$$

The actual head rise, $H_{\text{out}} - H_{\text{in}} = h_a$, is always less than the ideal head rise, h_i, by an amount equal to the head loss, h_L, in the pump. Some additional insight into the meaning of Eq. 12.13 can be obtained by using the following alternate version (see Eq. 12.8).

The pump actual head rise is less than the pump ideal head rise by an amount equal to the head loss in the pump.

$$h_i = \frac{1}{2g}[(V_2^2 - V_1^2) + (U_2^2 - U_1^2) + (W_1^2 - W_2^2)] \quad \textbf{(12.14)}$$

A detailed examination of the physical interpretation of Eq. 12.14 would reveal the following. The first term in brackets on the right-hand side represents the increase in the kinetic energy of the fluid, and the other two terms represent the pressure head rise that develops across the impeller due to the centrifugal effect, $U_2^2 - U_1^2$, and the diffusion of relative flow in the blade passages, $W_1^2 - W_2^2$.

An appropriate relationship between the flowrate and the pump ideal head rise can be obtained as follows. Often the fluid has no tangential component of velocity $V_{\theta 1}$, or *swirl*, as it enters the impeller; that is, the angle between the absolute velocity and the tangential direction is 90° ($\alpha_1 = 90°$ in Fig. 12.8). In this case, Eq. 12.13 reduces to

$$h_i = \frac{U_2V_{\theta 2}}{g} \quad \textbf{(12.15)}$$

From Fig. 12.8*c*

$$\cot \beta_2 = \frac{U_2 - V_{\theta 2}}{V_{r2}}$$

so that Eq. 12.15 can be expressed as

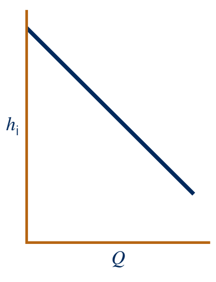
$$h_i = \frac{U_2^2}{g} - \frac{U_2V_{r2}\cot\beta_2}{g} \quad \textbf{(12.16)}$$

The flowrate, Q, is related to the radial component of the absolute velocity through the equation

$$Q = 2\pi r_2 b_2 V_{r2} \quad \textbf{(12.17)}$$

where b_2 is the impeller blade height at the radius r_2. Thus, combining Eqs. 12.16 and 12.17 yields

$$h_i = \frac{U_2^2}{g} - \frac{U_2\cot\beta_2}{2\pi r_2 b_2 g}Q \quad \textbf{(12.18)}$$



This equation is graphed in the margin and shows that the ideal or maximum head rise for a centrifugal pump varies linearly with Q for a given blade geometry and angular velocity. For actual pumps, the blade angle β_2 falls in the range of 15°–35°, with a normal range of $20° < \beta_2 < 25°$, and with $15° < \beta_1 < 50°$ (Ref. 10). Blades with $\beta_2 < 90°$ are called *backward curved*, whereas blades with $\beta_2 > 90°$ are called *forward curved.* Pumps are not usually designed with forward-curved vanes since such pumps tend to suffer unstable flow conditions.

EXAMPLE 12.2 Centrifugal Pump Performance Based on Inlet/Outlet Velocities

GIVEN Water is pumped at the rate of 5300 L/min through a centrifugal pump operating at a speed of 1750 rpm. The impeller has a uniform blade height, b, of 5 cm with $r_1 = 4$ cm and $r_2 = 18$ cm, and the exit blade angle β_2 is 23° (see Fig. 12.8). Assume ideal flow conditions and that the tangential velocity component, $V_{\theta 1}$, of the water entering the blade is zero ($\alpha_1 = 90°$).

FIND Determine **(a)** the tangential velocity component, $V_{\theta 2}$, at the exit, **(b)** the ideal head rise, h_i, and **(c)** the power, $\dot{W}_{\text{shaft}}$, transferred to the fluid. Discuss the difference between ideal and actual head rise. Is the power, $\dot{W}_{\text{shaft}}$, ideal or actual? Explain.

SOLUTION

(a) At the exit the velocity diagram is as shown in Fig. 12.8*c*, where $\mathbf{V}_2$ is the absolute velocity of the fluid, $\mathbf{W}_2$ is the relative velocity, and $\mathbf{U}_2$ is the tip velocity of the impeller with

$$U_2 = r_2\omega = (18{,}100 \text{ m})(2\pi \text{ rad/rev})\frac{(1750 \text{ rpm})}{(60 \text{ s/min})}$$
$$= 33 \text{ m/s}$$

Since the flowrate is given, it follows that

$$Q = 2\pi r_2 b_2 V_{r2}$$

or

$$V_{r2} = \frac{Q}{2\pi r_2 b_2}$$
$$= \frac{5300 \text{ L/min}}{(1000 \text{ L/m}^3)(60 \text{ s/min})(2\pi)(18/100 \text{ m})(5/100 \text{ m})}$$
$$= 1.6 \text{ m/s}$$

From Fig. 12.8*c* we see that

$$\cot \beta_2 = \frac{U_2 - V_{\theta 2}}{V_{r2}}$$

so that

$$V_{\theta 2} = U_2 - V_{r2}\cot\beta_2$$
$$= (33 - 1.6 \cot 23°) \text{ m/s}$$
$$= 29 \text{ m/s} \qquad \textbf{(Ans)}$$

(b) From Eq. 12.15 the ideal head rise is given by

$$h_i = \frac{U_2 V_{\theta 2}}{g} = \frac{(33 \text{ m/s})(29 \text{ m/s})}{9.8 \text{ m/s}^2}$$
$$= 98 \text{ m} \qquad \textbf{(Ans)}$$

Alternatively, from Eq. 12.16, the ideal head rise is

$$h_i = \frac{U_2^2}{g} - \frac{U_2 V_{r2}\cot\beta_2}{g}$$
$$= \frac{(33 \text{ m/s})^2}{9.8 \text{ m/s}^2} - \frac{(33 \text{ m/s})(1.6 \text{ m/s})\cot 23°}{9.8 \text{ m/s}^2}$$
$$= 98 \text{ m} \qquad \textbf{(Ans)}$$

(c) From Eq. 12.11, with $V_{\theta 1} = 0$, the power transferred to the fluid is given by the equation

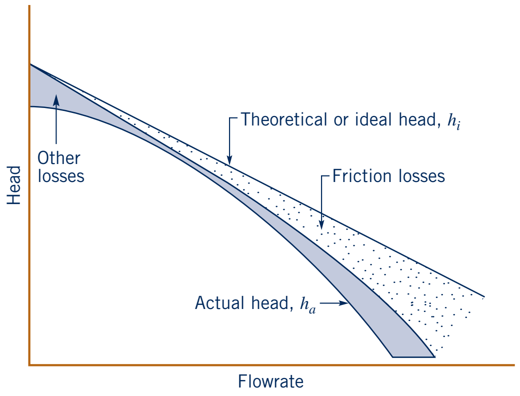
$$\dot{W}_{\text{shaft}} = \rho Q U_2 V_{\theta 2}$$
$$= \frac{(1000 \text{ kg/m}^3)(5300 \text{ L/min})(33 \text{ m/s})(29 \text{ m/s})}{[(\text{kg}\cdot\text{m/s}^2)/\text{N}](1000 \text{ L/m}^3)(60 \text{ s/min})}$$
$$= (84{,}500 \text{ m}\cdot\text{N/s})(1 \text{ W/N}\cdot\text{m/s}) = 84.5 \text{ kW} \qquad \textbf{(Ans)}$$

Note that the ideal head rise and the power transferred to the fluid are related through the relationship

$$\dot{W}_{\text{shaft}} = \rho g Q h_i$$

COMMENT It should be emphasized that results given in the previous equation involve the ideal head rise. The actual head-rise performance characteristics of a pump are usually determined by experimental measurements obtained in a testing laboratory. The actual head rise is always less than the ideal head rise for a specific flowrate because of the loss of available energy associated with actual flows. Also, it is important to note that even if actual values of U_2 and V_{r2} are used in Eq. 12.16, the ideal head rise is calculated. The only idealization used in this example problem is that the exit flow angle is identical to the blade angle at the exit. If the actual exit flow angle was made available in this example, it could have been used in Eq. 12.16 to calculate the ideal head rise.

The pump power, $\dot{W}_{\text{shaft}}$, is the actual power required to achieve a blade speed of 33 m/s, a flowrate of 5300 L/min, and the tangential velocity, $V_{\theta 2}$, associated with this example. If pump losses could somehow be reduced to zero (every pump designer's dream), the actual and ideal head rise would have been identical at 98 m. As is, the ideal head rise is 98 m and the actual head rise something less.



■ **Figure 12.9** **Effect of losses on the pump head–flowrate curve.**

Ideal and actual head-rise levels differ by the head loss.

Figure 12.9 shows the ideal head versus flowrate curve (Eq. 12.18) for a centrifugal pump with backward-curved vanes ($\beta_2 < 90°$). Since there are simplifying assumptions (i.e., zero losses) associated with the equation for h_i, we would expect that the actual rise in head of fluid, h_a, would be less than the ideal head rise, and this is indeed the case. As shown in Fig. 12.9, the h_a versus Q curve lies below the ideal head-rise curve and shows a nonlinear variation with Q. The differences between the two curves (as represented by the shaded areas between the curves) arise from several sources. These differences include losses due to fluid skin friction in the blade passages, which vary as Q^2, and other losses due to such factors as flow separation, impeller blade-casing clearance flows, and other three-dimensional flow effects. Near the design flowrate, some of these other losses are minimized.

Centrifugal pump design is a highly developed field, with much known about pump theory and design procedures (see, for example, Refs. 4–6). However, due to the general complexity of flow through a centrifugal pump, the actual performance of the pump cannot be accurately predicted on a completely theoretical basis as indicated by the data of Fig. 12.9. Actual pump performance is determined experimentally through tests on the pump. From these tests, pump characteristics are determined and presented as ***pump performance curves.*** It is this information that is most helpful to the engineer responsible for incorporating pumps into a given flow system.

12.4.2 Pump Performance Characteristics

The actual head rise, h_a, gained by fluid flowing through a pump can be determined with an experimental arrangement of the type shown in Fig. 12.10, using the energy equation (Eq. 5.84 with $h_a = h_s - h_L$ where h_s is the shaft work head and is identical to h_i, and h_L is the pump head loss)

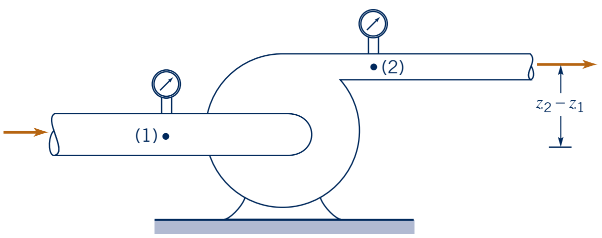
$$h_a = \frac{p_2 - p_1}{\gamma} + z_2 - z_1 + \frac{V_2^2 - V_1^2}{2g} \qquad \textbf{(12.19)}$$

with sections (1) and (2) at the pump inlet and exit, respectively. Typically, the differences in elevations and velocities are small so that

$$h_a \approx \frac{p_2 - p_1}{\gamma} \qquad \textbf{(12.20)}$$

The power, $\mathcal{P}_f$, gained by the fluid is given by the equation

$$\mathcal{P}_f = \gamma Q h_a \qquad \textbf{(12.21)}$$



■ **Figure 12.10** **Typical experimental arrangement for determining the head rise gained by a fluid flowing through a pump.**

and this quantity, expressed in terms of kilowatt is traditionally called the *water kilowatt*. Thus,

$$\mathcal{P}_f = \text{water kilowatt} = \gamma Q h_a \tag{12.23}$$

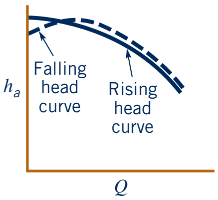
with γ expressed in N/m^3, Q in m^3/s, and h_a in m. Note that if the pumped fluid is not water, the γ appearing in Eq. 12.22 must be the specific weight of the fluid moving through the pump.

In addition to the head or power added to the fluid, the ***overall efficiency,*** η, is of interest, where

$$\eta = \frac{\text{power gained by the fluid}}{\text{shaft power driving the pump}} = \frac{\mathcal{P}_f}{\dot{W}_{\text{shaft}}}$$

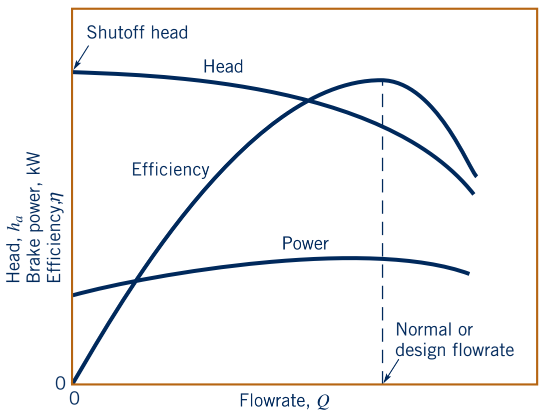
Overall pump efficiency is the ratio of power actually gained by the fluid to the shaft power supplied.

The overall pump efficiency is affected by the *hydraulic losses* in the pump, as previously discussed, and in addition, by the *mechanical losses* in the bearings and seals. There may also be some power loss due to leakage of the fluid between the back surface of the impeller hub plate and the casing, or through other pump components. This leakage contribution to the overall efficiency is called the *volumetric loss*. Thus, the overall efficiency arises from three sources, the *hydraulic efficiency*, η_h, the *mechanical efficiency,* η_m, and the *volumetric efficiency*, η_v, so that $\eta = \eta_h \eta_m \eta_v$.

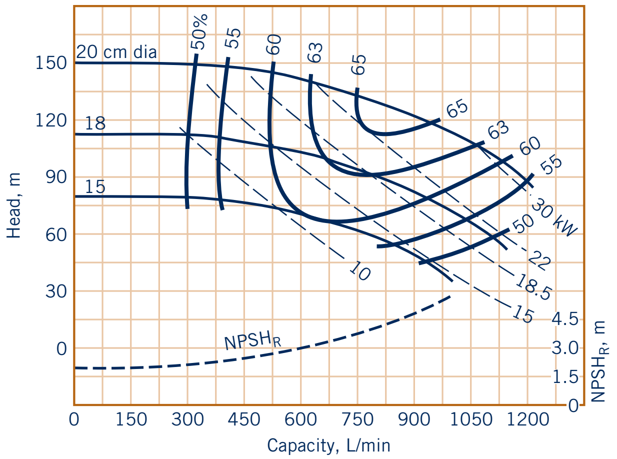


Performance characteristics for a given pump geometry and operating speed are usually given in the form of plots of h_a, η, and power versus Q (commonly referred to as *capacity*) as illustrated in Fig. 12.11. Actually, only two curves are needed since h_a, η, and power are related through Eq. 12.23. For convenience, all three curves are usually provided. Note that for the pump characterized by the data of Fig. 12.11, the head curve continuously rises as the flowrate decreases, and in this case the pump is said to have a *rising head* curve. As shown by the figure in the margin, pumps may also have $h_a - Q$ curves that initially rise as Q is decreased from the design value and then fall with a continued decrease in Q. These pumps have a *falling head* curve. The head developed by the pump at zero discharge is called the shutoff head, and it represents the rise in pressure head across the pump with the discharge valve closed. Since there is no flow with the valve closed, the related efficiency is zero, and the power supplied by the pump (power at $Q = 0$) is simply dissipated as heat. Although centrifugal pumps can be operated for short periods of time with the discharge valve closed, damage will occur due to overheating and large mechanical stress with any extended operation with the valve closed.

As can be seen from Fig. 12.11, as the discharge is increased from zero the power increases, with a subsequent fall as the maximum discharge is approached. As previously noted, with h_a and power known, the efficiency can be calculated. As shown in Fig. 12.11, the efficiency is a function of the flowrate and reaches a maximum value at some particular value of the flowrate, commonly referred to as the *normal* or *design* flowrate or capacity for the pump. The points on the various curves corresponding to the maximum efficiency are denoted as the *best efficiency points* (BEP). It is apparent that when selecting a pump for a particular application, it is usually desirable to have the pump operate near its maximum efficiency. Thus, performance curves of the type shown in Fig. 12.11 are very important to the engineer responsible for



■ **Figure 12.11** Typical performance characteristics for a centrifugal pump of a given size operating at a constant impeller speed.



■ **Figure 12.12** **Performance curves for a two-stage centrifugal pump operating at 3500 rpm. Data given for three different impeller diameters.**

the selection of pumps for a particular flow system. Matching the pump to a particular flow system is discussed in Section 12.4.4.

Pump performance characteristics are also presented in charts of the type shown in Fig. 12.12. Since impellers with different diameters may be used in a given casing, performance characteristics for several impeller diameters can be provided with corresponding lines of constant efficiency and power as illustrated in Fig. 12.12. Thus, the same information can be obtained from this type of graph as from the curves shown in Fig. 12.11.

It is to be noted that an additional curve is given in Fig. 12.12, labeled NPSH$_R$, which stands for *required net positive suction head*. As discussed in the following section, the significance of this curve is related to conditions on the suction side of the pump, which must also be carefully considered when selecting and positioning a pump.

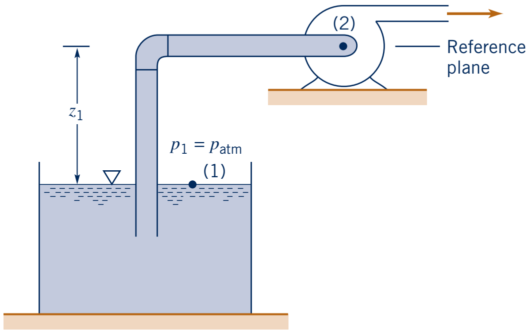
12.4.3 Net Positive Suction Head (NPSH)

Cavitation, which may occur when pumping a liquid, is usually avoided.

On the suction side of a pump, low pressures are commonly encountered, with the concomitant possibility of cavitation occurring within the pump. As discussed in Section 1.8, cavitation occurs when the liquid pressure at a given location is reduced to the vapor pressure of the liquid. When this occurs, vapor bubbles form (the liquid starts to "boil"); this phenomenon can cause a loss in efficiency as well as structural damage to the pump. To characterize the potential for cavitation, the difference between the total head on the suction side, near the pump impeller inlet, $p_s/\gamma + V_s^2/2g$, and the liquid vapor pressure head, p_v/γ, is used. The position reference for the elevation head passes through the centerline of the pump impeller inlet. This difference is called the net positive suction head (NPSH) so that

$$\text{NPSH} = \frac{p_s}{\gamma} + \frac{V_s^2}{2g} - \frac{p_v}{\gamma} \qquad \textbf{(12.24)}$$

There are actually two values of NPSH of interest. The first is the *required* NPSH, denoted NPSH$_R$, that must be maintained, or exceeded, so that cavitation will not occur. Since pressures lower than those in the suction pipe will develop in the impeller eye, it is usually necessary to determine experimentally, for a given pump, the required NPSH$_R$. This is the curve shown in Fig. 12.12. Pumps are tested to determine the value for NPSH$_R$, as defined by Eq. 12.24, by either directly detecting cavitation or observing a change in the head–flowrate curve (Ref. 7). The second value for NPSH of concern is the *available* NPSH, denoted NPSH$_A$, which represents the head that actually occurs for the particular flow system. This value can be determined experimentally, or calculated if the system parameters are known. For example, a typical flow system is shown in Fig. 12.13. The energy equation applied between the free liquid surface,



■ **Figure 12.13** Schematic of a pump installation in which the pump must lift fluid from one level to another.

where the pressure is atmospheric, p_{atm}, and a point on the suction side of the pump near the impeller inlet yields

$$\frac{p_{atm}}{\gamma} - z_1 = \frac{p_s}{\gamma} + \frac{V_s^2}{2g} + \sum h_L$$

where Σh_L represents head losses between the free surface and the pump impeller inlet. Thus, the head available at the pump impeller inlet is

$$\frac{p_s}{\gamma} + \frac{V_s^2}{2g} = \frac{p_{atm}}{\gamma} - z_1 - \sum h_L$$

so that

$$\text{NPSH}_A = \frac{p_{atm}}{\gamma} - z_1 - \sum h_L - \frac{p_v}{\gamma} \quad \textbf{(12.25)}$$

For this calculation, absolute pressures are normally used since the vapor pressure is usually specified as an absolute pressure. For proper pump operation it is necessary that

For proper pump operation, the available net positive suction head must be greater than the required net positive suction head.

$$\text{NPSH}_A \geq \text{NPSH}_R$$

It is noted from Eq. 12.25 that as the height of the pump impeller above the fluid surface, z_1, is increased, the NPSH_A is decreased. Therefore, there is some critical value for z_1 above which the pump cannot operate without cavitation. The specific value depends on the head losses and the value of the vapor pressure. It is further noted that if the supply tank or reservoir is *above* the pump, z_1 will be negative in Eq. 12.25, and the NPSH_A will increase as this height is increased.

EXAMPLE 12.3 Net Positive Suction Head

GIVEN A centrifugal pump is to be placed above a large, open water tank, as shown in Fig. 12.13, and is to pump water at a rate of 1.4×10^{-2} m^3/s. At this flowrate the required net positive suction head, NPSH_R, is 4.5 m, as specified by the pump manufacturer. The water temperature is 30 °C and atmospheric pressure is 101.3 kPa. Assume that the major head loss between the tank and the pump inlet is due to filter at the pipe inlet having a minor loss coefficient $K_L = 20$. Other losses can be neglected. The pipe on the suction side of the pump has a diameter of 10 cm.

FIND Determine the maximum height, z_1, that the pump can be located above the water surface without cavitation. If you were required to place a valve in the flow path, would you place it upstream or downstream of the pump? Why?

SOLUTION

From Eq. 12.25 the available net positive suction head, NPSH_A, is given by the equation

$$\text{NPSH}_A = \frac{p_{atm}}{\gamma} - z_1 - \sum h_L - \frac{p_v}{\gamma}$$

and the maximum value for z_1 will occur when $\text{NPSH}_A = \text{NPSH}_R$. Thus,

$$(z_1)_{max} = \frac{p_{atm}}{\gamma} - \sum h_L - \frac{p_v}{\gamma} - \text{NPSH}_R \quad \textbf{(1)}$$

Since the only head loss to be considered is the loss

$$\sum h_L = K_L \frac{V^2}{2g}$$

with

$$V = \frac{Q}{A} = \frac{1.4 \times 10^{-2}\ \text{m}^3/\text{s}}{(\pi/4)(10/100\ \text{cm})^2} = 1.8\ \text{m/s}$$

it follows that

$$\sum h_L = \frac{(20)(1.8\ \text{m/s})^2}{2(9.8\ \text{m/s}^2)} = 3\ \text{m}$$

From Table B.1 the water vapor pressure at 30 °C is 4.243 kPa and $\gamma = 9.765\ \text{N/m}^3$. Equation (1) can now be written as

$$(z_1)_{\text{max}} = \frac{(101.3\ \text{kPa})}{9.765\ \text{N/m}^3} - 3\ \text{m} - \frac{4.243\ \text{kPa}}{9.765\ \text{N/m}^3} - 4.5\ \text{m}$$
$$= 2.3\ \text{m} \qquad \textbf{(Ans)}$$

Thus, to prevent cavitation, with its accompanying poor pump performance, the pump should not be located higher than 2.3 m above the water surface.

COMMENT If the valve is placed upstream of the pump, not only would the pump have to operate with an additional loss in the system, it would now operate with a lower inlet pressure because of this additional upstream loss and could now suffer cavitation with its usually negative consequences. If the valve is placed downstream of the pump, the pump would need to operate with more loss in the system and with higher back pressure than without the valve. Depending on the stability of the pump at higher back pressures, this could be inconsequential or important. Usually, pumps are stable even with higher back pressures. So, placing the valve on the downstream side of the pump is normally the better choice.

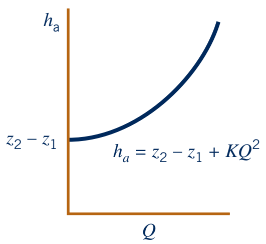
12.4.4 System Characteristics and Pump Selection

The system equation relates the actual head gained by the fluid to the flowrate.

A typical flow system in which a pump is used is shown in Fig. 12.14. The energy equation applied between points (1) and (2) indicates that

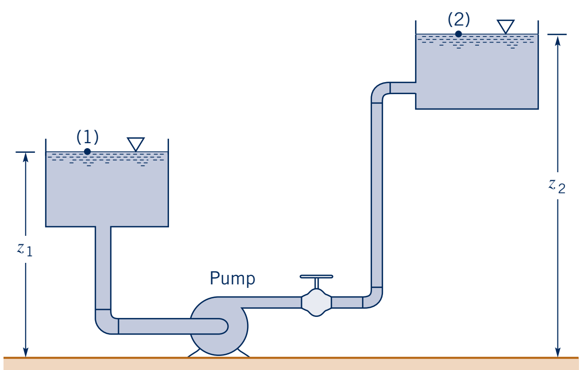
$$h_a = z_2 - z_1 + \sum h_L \qquad \textbf{(12.26)}$$

where h_a is the actual head gained by the fluid from the pump, and Σh_L represents all friction losses in the pipe and minor losses for pipe fittings and valves. From our study of pipe flow, we know that typically h_L varies approximately as the flowrate squared; that is, $h_L \propto Q^2$ (see Section 8.4). Thus, Eq. 12.26 can be written in the form

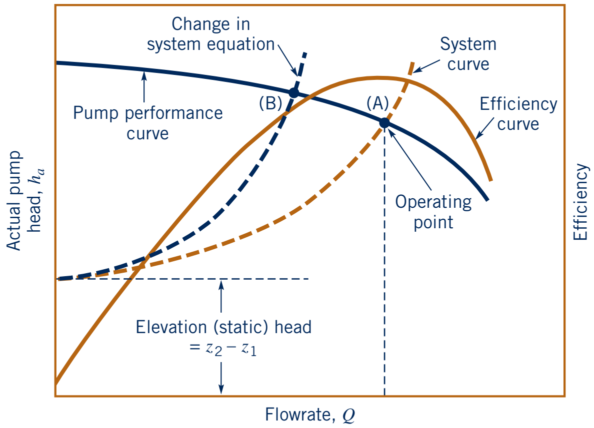


$$h_a = z_2 - z_1 + KQ^2 \qquad \textbf{(12.27)}$$

where K depends on the pipe sizes and lengths, friction factors, and minor loss coefficients. Equation 12.27, which is shown in the figure in the margin, is the ***system equation*** and shows how the actual head gained by the fluid from the pump is related to the system parameters. In this case the parameters include the change in elevation head, $z_2 - z_1$, and the losses due to friction as expressed by KQ^2. Each flow system has its own specific system equation. If the flow is laminar, the frictional losses will be proportional to Q rather than Q^2 (see Section 8.2).



■ **Figure 12.14** **Typical flow system.**



■ **Figure 12.15** **Utilization of the system curve and the pump performance curve to obtain the operating point for the system.**

The intersection of the pump performance curve and the system curve is the operating point.

There is also a unique relationship between the actual pump head gained by the fluid and the flowrate, which is governed by the pump design (as indicated by the pump performance curve). To select a pump for a particular application, it is necessary to utilize both the *system curve*, as determined by the system equation, and the pump performance curve. If both curves are plotted on the same graph, as illustrated in Fig. 12.15, their intersection (point A) represents the operating point for the system. That is, this point gives the head and flowrate that satisfy both the system equation and the pump equation. On the same graph the pump efficiency is shown. Ideally, we want the operating point to be near the best efficiency point (BEP) for the pump. For a given pump, it is clear that as the system equation changes, the operating point will shift. For example, if the pipe friction increases due to pipe wall fouling, the system curve changes, resulting in the operating point A shifting to point B in Fig. 12.15 with a reduction in flowrate and efficiency. The following example shows how the system and pump characteristics can be used to decide if a particular pump is suitable for a given application.

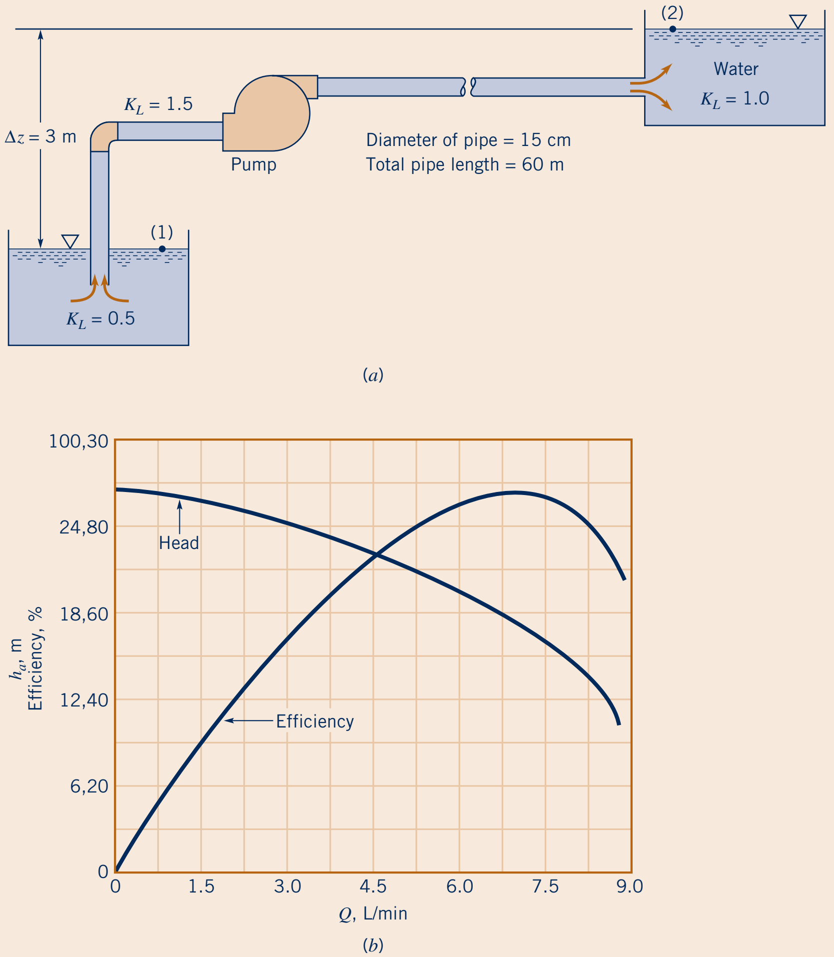
Fluids in the News

Space Shuttle fuel pumps The fuel *pump* of your car engine is vital to its operation. Similarly, the fuels (liquid hydrogen and oxygen) of each Space Shuttle main engine (there are three per shuttle) rely on multistage *turbopumps* to get from storage tanks to main combustors. High pressures are utilized throughout the pumps to avoid cavitation. The pumps, some *centrifugal* and some *axial*, are driven by axial-flow, *multi-stage turbines*. Pump speeds are as high as 35,360 rpm. The liquid oxygen is pumped from 0.7 to 51 MPa, the liquid hydrogen from 0.21 to 50 MPa. Liquid hydrogen and oxygen flowrates of about 65 kL/min and 23 kL/min, respectively, are achieved. These pumps could empty your home swimming pool in seconds. The hydrogen goes from −252.7 °C in storage to 3315.5 °C in the combustion chamber!

EXAMPLE 12.4 Use of Pump Performance Curves

GIVEN Water is to be pumped from one large, open tank to a second large, open tank as shown in Fig. E12.4*a*. The pipe diameter throughout is 15 cm, and the total length of the pipe between the pipe entrance and exit is 60 m. Minor loss coefficients for the entrance, exit, and the elbow are shown, and the friction factor for the pipe can be assumed constant and equal to 0.02. A certain centrifugal pump having the performance characteristics shown in Fig. E12.4*b* is suggested as a good pump for this flow system.

FIND With this pump, what would be the flowrate between the tanks? Do you think this pump would be a good choice?



■ **Figure E12.4*a, b***

Solution

Application of the energy equation between the two free surfaces, points (1) and (2) as indicated, gives

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_a = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + f\frac{\ell}{D}\frac{V^2}{2g} + \sum K_L \frac{V^2}{2g} \quad \textbf{(1)}$$

Thus, with $p_1 = p_2 = 0$, $V_1 = V_2 = 0$, $\Delta z = z_2 - z_1 = 3$ m, $f = 0.02$, $D = 15/100$ m, and $\ell = 60$ m, Eq. 1 becomes

$$h_a = 3 + \left[0.02\frac{(60 \text{ m})}{(15/100 \text{ m})} + (0.5 + 1.5 + 1.0)\right]\frac{V^2}{2(9.8 \text{ m/s}^2)} \quad \textbf{(2)}$$

where the given minor loss coefficients have been used. Since

$$V = \frac{Q}{A} = \frac{Q(\text{m}^3/\text{s})}{(\pi/4)(15/100 \text{ m})^2}$$

Eq. 2 can be expressed as

$$h_a = 3 + 1795\, Q^2 \quad \textbf{(3)}$$

where Q is in m^3/s, or with Q in liters per minute

$$h_a = 3 + 5 \times 10^{-7} Q^2 \quad \textbf{(4)}$$

Equation 3 or 4 represents the system equation for this particular flow system and reveals how much actual head the fluid will need to gain from the pump to maintain a certain flowrate. Performance data shown in Fig. E12.4*b* indicate the actual head the fluid will gain from this particular pump when it operates at a certain flowrate. Thus, when Eq. 4 is plotted on the same graph with performance data, the intersection of the two curves represents the operating point for the pump and the system. This combination is shown in Fig. E12.4*c* with the intersection (as obtained graphically) occurring at

$$Q = 6 \text{ kL/min} \quad \textbf{(Ans)}$$

with the corresponding actual head gained equal to 21 m.

Another concern is whether the pump is operating efficiently at the operating point. As can be seen from Fig. E12.4*c*, although this is not peak efficiency, which is about 86%, it is close (about 84%). Thus, this pump would be a satisfactory choice, assuming the 6 kL/min flowrate is at or near the desired flowrate.

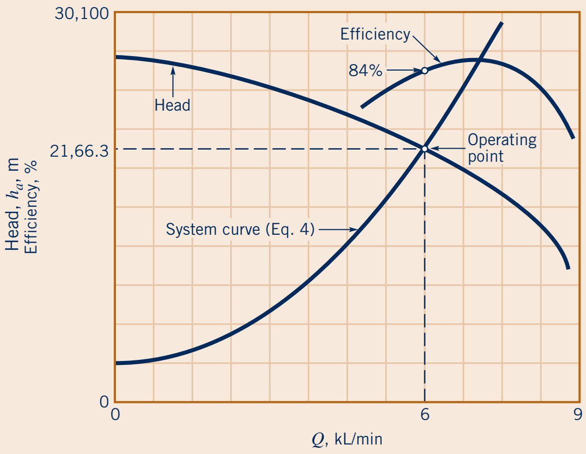


Figure E12.4*c* ***(Continued)***

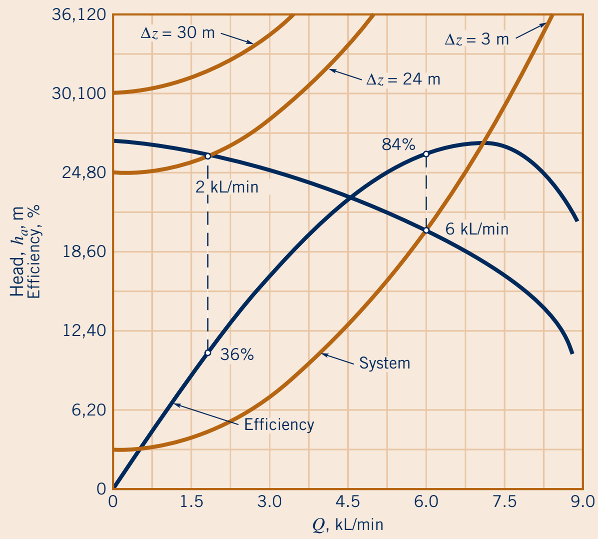


Figure E12.4*d*

The amount of pump head needed at the pump shaft is 21 m/0.84 = 25 m. The power needed to drive the pump is

$$\dot{W}_{\text{shaft}} = \frac{\gamma Q h_a}{\eta}$$

$$= \frac{(9800\ \text{N/m}^3)[(6\ \text{kL/min})/(1000\ \text{L/m}^3)(60\ \text{s/min})](21\ \text{m})}{0.84}$$

$$= 24{,}500\ \text{m} \cdot \text{N/s} = 24.5\ \text{kW}$$

COMMENT By repeating the calculations for $\Delta z = z_2 - z_1 = 24$ m and 30 m (rather than the given 3 m), the results shown in Fig. E12.4*d* are obtained. Although the given pump could be used with $\Delta z = 30$ m (provided that the 2 kL/min flowrate produced is acceptable), it would not be an ideal pump for this application since its efficiency would be only 36%. Energy could be saved by using a different pump with a performance curve that more nearly matches the new system requirements (i.e., higher efficiency at the operating condition). On the other hand, the given pump would not work at all for $\Delta z = 30$ m since its maximum head ($h_a = 26.8$ m when $Q = 0$) is not enough to lift the water 30 m, let alone overcome head losses. This is shown in Fig. E12.4*d* by the fact that for $\Delta z = 30$ m the system curve and the pump performance curve do not intersect.

Note that head loss within the pump itself was accounted for with the pump efficiency, η. Thus, $h_s = h_a/\eta$, where h_s is the pump shaft work head and h_a is the actual head rise experienced by the flowing fluid.

For two pumps in series, add heads; for two in parallel, add flowrates.

Pumps can be arranged in series or in parallel to provide for additional head or flow capacity. When two pumps are placed in *series*, the resulting pump performance curve is obtained by adding heads at the same flowrate. As illustrated in Fig. 12.16*a*, for two identical pumps in series, both the actual head gained by the fluid and the flowrate are increased, but neither will be doubled if the system curve remains the same. The operating point is at (A) for one pump and moves to (B) for two pumps in series. For two identical pumps in *parallel*, the combined performance curve is obtained by adding flowrates at the same head, as shown in Fig. 12.16*b*. As illustrated, the flowrate for the system will not be doubled with the addition of two pumps in parallel (if the same system

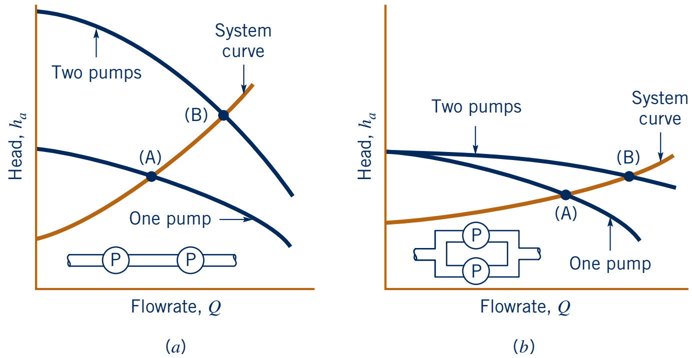


Figure 12.16 **Effect of operating pumps in (*a*) series and (*b*) in parallel.**

curve applies). However, for a relatively flat system curve, as shown in Fig. 12.16*b*, a significant increase in flowrate can be obtained as the operating point moves from point (A) to point (B).

12.5 Dimensionless Parameters and Similarity Laws

As discussed in Chapter 7, dimensional analysis is particularly useful in the planning and execution of experiments. Since the characteristics of pumps are usually determined experimentally, it is expected that dimensional analysis and similitude considerations will prove to be useful in the study and documentation of these characteristics.

From the previous section we know that the principal, dependent pump variables are the actual head rise, h_a, shaft power, $\dot{W}_{\text{shaft}}$, and efficiency, η. We expect that these variables will depend on the geometrical configuration, which can be represented by some characteristic diameter, D, other pertinent lengths, ℓ_i, and surface roughness, ε. In addition, the other important variables are flowrate, Q, the pump shaft rotational speed, ω, fluid viscosity, μ, and fluid density, ρ. We will only consider incompressible fluids presently, so compressibility effects need not concern us yet. Thus, any one of the dependent variables h_a, $\dot{W}_{\text{shaft}}$, and η can be expressed as

$$\text{Dependent variable} = f(D, \ell_i, \varepsilon, Q, \omega, \mu, \rho)$$

and a straightforward application of dimensional analysis leads to

$$\text{Dependent pi term} = \phi\left(\frac{\ell_i}{D}, \frac{\varepsilon}{D}, \frac{Q}{\omega D^3}, \frac{\rho \omega D^2}{\mu}\right) \tag{12.28}$$

Dimensionless pi terms and similarity laws are important pump considerations.

The dependent pi term involving the head is usually expressed as $C_H = gh_a/\omega^2 D^2$, where gh_a is the actual head rise in terms of energy per unit mass, rather than simply h_a, which is energy per unit weight. This dimensionless parameter is called the ***head rise coefficient.*** The dependent pi term involving the shaft power is expressed as $C_{\mathcal{P}} = \dot{W}_{\text{shaft}}/\rho\omega^3 D^5$, and this standard dimensionless parameter is termed the ***power coefficient.*** The rotational speed, ω, which appears in these dimensionless groups is expressed in rad/s. The final dependent pi term is the efficiency, η, which is already dimensionless. Thus, in terms of dimensionless parameters the performance characteristics are expressed as

$$C_H = \frac{gh_a}{\omega^2 D^2} = \phi_1\left(\frac{\ell_i}{D}, \frac{\varepsilon}{D}, \frac{Q}{\omega D^3}, \frac{\rho \omega D^2}{\mu}\right)$$

$$C_{\mathcal{P}} = \frac{\dot{W}_{\text{shaft}}}{\rho \omega^3 D^5} = \phi_2\left(\frac{\ell_i}{D}, \frac{\varepsilon}{D}, \frac{Q}{\omega D^3}, \frac{\rho \omega D^2}{\mu}\right)$$

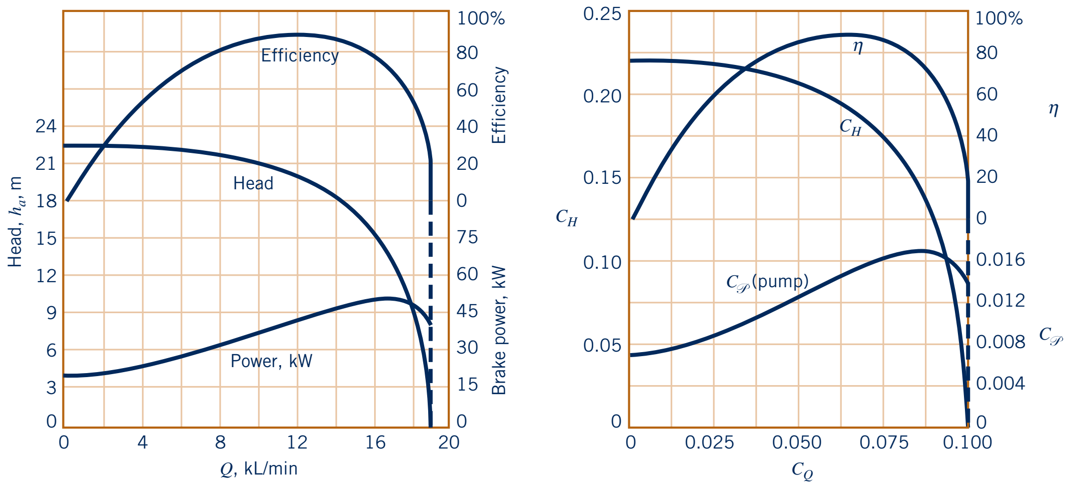
$$\eta = \frac{\rho g Q h_a}{\dot{W}_{\text{shaft}}} = \phi_3\left(\frac{\ell_i}{D}, \frac{\varepsilon}{D}, \frac{Q}{\omega D^3}, \frac{\rho \omega D^2}{\mu}\right)$$

The last pi term in each of the above equations is a form of Reynolds number that represents the relative influence of viscous effects. When the pump flow involves high Reynolds numbers, as is usually the case, experience has shown that the effect of the Reynolds number can be neglected. For simplicity, the relative roughness, ε/D, can also be neglected in pumps since the highly irregular shape of the pump chamber is usually the dominant geometric factor rather than the surface roughness. Thus, with these simplifications and for *geometrically similar* pumps (all pertinent dimensions, ℓ_i, scaled by a common length scale), the dependent pi terms are functions of only $Q/\omega D^3$, so that

$$\frac{gh_a}{\omega^2 D^2} = \phi_1\left(\frac{Q}{\omega D^3}\right) \tag{12.29}$$

$$\frac{\dot{W}_{\text{shaft}}}{\rho \omega^3 D^5} = \phi_2\left(\frac{Q}{\omega D^3}\right) \tag{12.30}$$

$$\eta = \phi_3\left(\frac{Q}{\omega D^3}\right) \tag{12.31}$$



(*a*) (*b*)

■ **Figure 12.17** **Typical performance data for a centrifugal pump: (*a*) characteristic curves for a 30.5 cm. centrifugal pump operating at 1000 rpm, (*b*) dimensionless characteristic curves. (Data from Ref. 8, used by permission.)**

The dimensionless parameter $C_Q = Q/\omega D^3$ is called the ***flow coefficient***. These three equations provide the desired similarity relationships among a family of geometrically similar pumps. If two pumps from the family are operated at the same value of flow coefficient

$$\left(\frac{Q}{\omega D^3}\right)_1 = \left(\frac{Q}{\omega D^3}\right)_2 \tag{12.32}$$

it then follows that

$$\left(\frac{gh_a}{\omega^2 D^2}\right)_1 = \left(\frac{gh_a}{\omega^2 D^2}\right)_2 \tag{12.33}$$

$$\left(\frac{\dot{W}_{\text{shaft}}}{\rho\omega^3 D^5}\right)_1 = \left(\frac{\dot{W}_{\text{shaft}}}{\rho\omega^3 D^5}\right)_2 \tag{12.34}$$

$$\eta_1 = \eta_2 \tag{12.35}$$

where the subscripts 1 and 2 refer to any two pumps from the family of geometrically similar pumps.

Pump scaling laws relate geometrically similar pumps.

With these so-called ***pump scaling laws*** it is possible to experimentally determine the performance characteristics of one pump in the laboratory and then use these data to predict the corresponding characteristics for other pumps within the family under different operating conditions. Figure 12.17*a* shows some typical curves obtained for a centrifugal pump. Figure 12.17*b* shows the results plotted in terms of the dimensionless coefficients, C_Q, C_H, $C_{\mathcal{P}}$, and η. From these curves the performance of different-sized, geometrically similar pumps can be predicted, as can the effect of changing speeds on the performance of the pump from which the curves were obtained. It is to be noted that the efficiency, η, is related to the other coefficients through the relationship $\eta = C_Q C_H C_{\mathcal{P}}^{-1}$. This follows directly from the definition of η.

EXAMPLE 12.5 Use of Pump Scaling Laws

GIVEN A 20 cm diameter centrifugal pump operating at 1200 rpm is geometrically similar to the 30 cm diameter pump having the performance characteristics of Figs. 12.17*a* and 12.17*b* while operating at 1000 rpm. The working fluid is water at 15 °C.

FIND For peak efficiency, predict the discharge, actual head rise, and shaft power (kW) for this smaller pump.

Solution

As is indicated by Eq. 12.31, for a given efficiency the flow coefficient has the same value for a given family of pumps. From Fig. 12.17*b* we see that at peak efficiency $C_Q = 0.0625$. Thus, for the 20 cm pump

$$Q = C_Q \omega D^3$$
$$= (0.0625)(1200/60 \text{ rev/s})(2\pi \text{ rad/rev})(20/100 \text{ m})^3$$
$$Q = 0.063 \text{ m}^3/\text{s} \qquad \textbf{(Ans)}$$

or in terms of liter/min

$$Q = (0.063 \text{ m}^3/\text{s})(1000 \text{ L/m}^3)(60 \text{ s/min})$$
$$= 4 \text{ kL/min} \qquad \textbf{(Ans)}$$

The actual head rise and the shaft power (kW) can be determined in a similar manner since at peak efficiency $C_H = 0.19$ and $C_{\mathcal{P}} = 0.014$, so that with $\omega = 1200$ rev/min (1 min/60 s) (2π rad/rev) = 126 rad/s

$$h_a = \frac{C_H \omega^2 D^2}{g} = \frac{(0.19)(126 \text{ rad/s})^2(20/100 \text{ m})^2}{9.8 \text{ m/s}^2} = 12.3 \text{ m} \qquad \textbf{(Ans)}$$

and

$$\dot{W}_{\text{shaft}} = C_{\mathcal{P}} \rho \omega^3 D^5$$
$$= (0.014)(1000 \text{ kg/m}^3)(126 \text{ rad/s})^3(20/100 \text{ m})^5$$
$$= 9.0 \text{ kN} \cdot \text{m/s}$$
$$\dot{W}_{\text{shaft}} = \frac{9.0 \text{ kN} \cdot \text{m/s}}{1.0 \text{ kN} \cdot \text{m/s}} = 9.0 \text{ kW} \qquad \textbf{(Ans)}$$

COMMENT This last result gives the shaft power (kW), which is the power supplied to the pump shaft. The power actually gained by the fluid is equal to $\gamma Q h_a$, which in this example is

$$\mathcal{P}_f = \gamma Q h_a = (9800 \text{ N/m}^2)(0.063 \text{ m}^3/\text{s})(12.3 \text{ m}) = 7.6 \text{ kN} \cdot \text{m/s}$$

Thus, the efficiency, η, is

$$\eta = \frac{\mathcal{P}_f}{\dot{W}_{\text{shaft}}} = \frac{7.6}{9} = 84\%$$

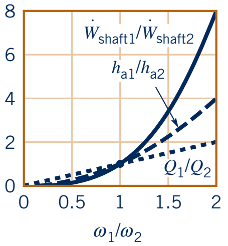
which checks with the efficiency curve of Fig. 12.17*b*.

12.5.1 Special Pump Scaling Laws

Effects of changes in pump operating speed and impeller diameter are often of interest.

Two special cases related to pump similitude commonly arise. In the first case we are interested in how a change in the operating speed, ω, for a *given pump*, affects pump characteristics. It follows from Eq. 12.32 that for the same flow coefficient (and therefore the same efficiency) with $D_1 = D_2$ (the same pump)

$$\frac{Q_1}{Q_2} = \frac{\omega_1}{\omega_2} \tag{12.36}$$



The subscripts 1 and 2 now refer to the same pump operating at two different speeds at the same flow coefficient. Also, from Eqs. 12.33 and 12.34 it follows that

$$\frac{h_{a1}}{h_{a2}} = \frac{\omega_1^2}{\omega_2^2} \tag{12.37}$$

and

$$\frac{\dot{W}_{\text{shaft1}}}{\dot{W}_{\text{shaft2}}} = \frac{\omega_1^3}{\omega_2^3} \tag{12.38}$$

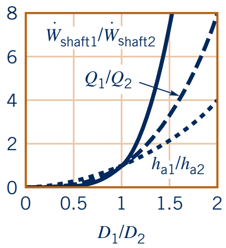
Thus, for a given pump operating at a given flow coefficient, the flow varies directly with speed, the head varies as the speed squared, and the power varies as the speed cubed. These effects of angular velocity variation are illustrated in the sketch in the margin. These scaling laws are useful in estimating the effect of changing pump speed when some data are available from a pump test obtained by operating the pump at a particular speed.

In the second special case we are interested in how a change in the impeller diameter, D, of a geometrically similar family of pumps, operating at a *given speed*, affects pump characteristics. As before, it follows from Eq. 12.32 that for the same flow coefficient with $\omega_1 = \omega_2$

$$\frac{Q_1}{Q_2} = \frac{D_1^3}{D_2^3} \tag{12.39}$$

Similarly, from Eqs. 12.33 and 12.34

$$\frac{h_{a1}}{h_{a2}} = \frac{D_1^2}{D_2^2} \tag{12.40}$$



Pump affinity laws relate the same pump at different speeds or geometrically similar pumps at the same speed.

and

$$\frac{\dot{W}_{shaft1}}{\dot{W}_{shaft2}} = \frac{D_1^5}{D_2^5} \tag{12.41}$$

Thus, for a family of geometrically similar pumps operating at a given speed and the same flow coefficient, the flow varies as the diameter cubed, the head varies as the diameter squared, and the power varies as the diameter raised to the fifth power. These strong effects of diameter variation are illustrated in the sketch in the margin. These scaling relationships are based on the condition that, as the impeller diameter is changed, all other important geometric variables are properly scaled to maintain geometric similarity. This type of geometric scaling is not always possible due to practical difficulties associated with manufacturing the pumps. It is common practice for manufacturers to put impellers of different diameters in the same pump casing. In this case, complete geometric similarity is not maintained, and the scaling relationships expressed in Eqs. 12.39, 12.40, and 12.41 will not, in general, be valid. However, experience has shown that if the impeller diameter change is not too large, less than about 20%, these scaling relationships can still be used to estimate the effect of a change in the impeller diameter. The pump similarity laws expressed by Eqs. 12.36 through 12.41 are sometimes referred to as the *pump affinity laws.*

The effects of viscosity and surface roughness have been neglected in the foregoing similarity relationships. However, it has been found that as the pump size decreases, these effects more significantly influence efficiency because of smaller clearances and blade size. An approximate, empirical relationship to estimate the influence of diminishing size on efficiency is (Ref. 9)

$$\frac{1 - \eta_2}{1 - \eta_1} \approx \left(\frac{D_1}{D_2}\right)^{1/5} \tag{12.42}$$

In general, it is to be expected that the similarity laws will not be very accurate if tests on a model pump with water are used to predict the performance of a prototype pump with a highly viscous fluid, such as oil, because at the much smaller Reynolds number associated with the oil flow, the fluid physics involved is different from the higher Reynolds number flow associated with water.

12.5.2 Specific Speed

A useful pi term can be obtained by eliminating diameter D between the flow coefficient and the head rise coefficient. This is accomplished by raising the flow coefficient to an appropriate exponent (1/2) and dividing this result by the head coefficient raised to another appropriate exponent (3/4) so that

$$\frac{(Q/\omega D^3)^{1/2}}{(gh_a/\omega^2 D^2)^{3/4}} = \frac{\omega\sqrt{Q}}{(gh_a)^{3/4}} = N_s \tag{12.43}$$

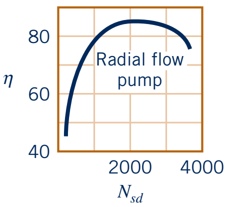
The dimensionless parameter N_s is called the ***specific speed***. Specific speed varies with flow coefficient just as the other coefficients and efficiency discussed earlier do. However, for any pump it is customary to specify a value of specific speed at the flow coefficient corresponding to peak efficiency only. For pumps with low Q and high h_a, the specific speed is low compared to a pump with high Q and low h_a. Centrifugal pumps typically are low-capacity, high-head pumps, and therefore have low specific speeds.

Specific speed as defined by Eq. 12.43 is dimensionless, and therefore independent of the system of units used in its evaluation as long as a consistent unit system is used. However, in the United States a modified, dimensional form of specific speed, N_{sd}, is commonly used, where

$$N_{sd} = \frac{\omega(\text{rpm})\sqrt{Q(\text{liter/min})}}{[h_a(\text{m})]^{3/4}} \tag{12.44}$$

In this case N_{sd} is said to be expressed in *U.S. customary units*. Typical values of N_{sd} are in the range $500 < N_{sd} < 4000$ for centrifugal pumps. Both N_s and N_{sd} have the same physical meaning, but their magnitudes will differ by a constant conversion factor ($N_{sd} = 2733\, N_s$) when ω in Eq. 12.43 is expressed in rad/s.

Each family or class of pumps has a particular range of values of specific speed associated with it. Thus, pumps that have low-capacity, high-head characteristics will have specific speeds that are



smaller than pumps that have high-capacity, low-head characteristics. The concept of specific speed is very useful to engineers and designers, since if the required head, flowrate, and speed are specified, it is possible to select an appropriate (most efficient) type of pump for a particular application. For example, as shown by the figure in the margin, as the specific speed, N_{sd}, increases beyond about 2000 the peak efficiency, η, of the purely radial-flow centrifugal pump starts to fall off, and other types of more efficient pump design are preferred. In addition to the centrifugal pump, the *axial-flow* pump is widely used. As discussed in Section 12.6, in an axial-flow pump the direction of flow is primarily parallel to the rotating shaft rather than radial as in the centrifugal pump. Axial-flow pumps are essentially high-capacity, low-head pumps, and therefore have large specific speeds ($N_{sd} > 9000$) compared to centrifugal pumps. *Mixed-flow* pumps combine features of both radial-flow and axial-flow pumps and have intermediate values of specific speed. Figure 12.18 illustrates how the specific speed changes as the configuration of the pump changes from centrifugal or radial to axial.

12.5.3 Suction Specific Speed

With an analysis similar to that used to obtain the specific speed pi term, the *suction specific speed*, S_s, can be expressed as

$$S_s = \frac{\omega\sqrt{Q}}{[g(\text{NPSH}_\text{R})]^{3/4}} \tag{12.45}$$

Specific speed may be used to approximate what general pump geometry (axial, mixed, or radial) to use for maximum efficiency.

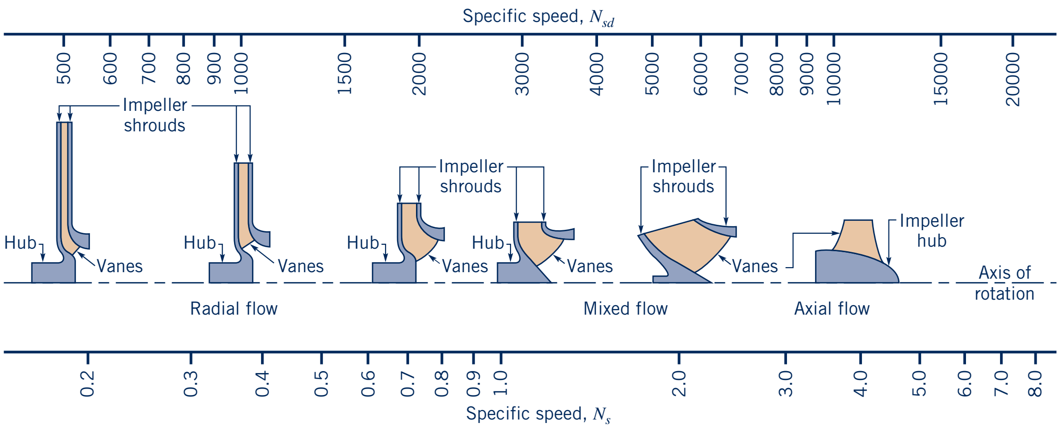
where h_a in Eq. 12.43 has been replaced by the required net positive suction head (NPSH_R). This dimensionless parameter is useful in determining the required operating conditions on the suction side of the pump. As was true for the specific speed, N_s, the value for S_s commonly used is for peak efficiency. For a family of geometrically similar pumps, S_s should have a fixed value. If this value is known, then the NPSH_R can be estimated for other pumps within the same family operating at different values of ω and Q.

As noted for N_s, the suction specific speed as defined by Eq. 12.45 is also dimensionless, and the value for S_s is independent of the system of units used. However, as was the case for specific speed, in the United States a modified dimensional form for the suction specific speed, designated as S_{sd}, is commonly used, where

$$S_{sd} = \frac{\omega(\text{rpm})\sqrt{Q(\text{L/min})}}{[\text{NPSH}_\text{R}\ (\text{m})]^{3/4}} \tag{12.46}$$

For double-suction pumps the discharge, Q, in Eq. 12.46 is one-half the total discharge.

Typical values for S_{sd} fall in the range of 7000 to 12,000 (Ref. 11). If S_{sd} is specified, Eq. 12.46 can be used to estimate the NPSH_R for a given set of operating conditions. However, this calculation would generally only provide an approximate value for the NPSH_R, and the actual determination of the NPSH_R for a particular pump should be made through a direct measurement whenever possible. Note that $S_{sd} = 2733\ S_s$, with ω expressed in rad/s in Eq. 12.45.



■ **Figure 12.18** **Variation in specific speed at maximum efficiency with type of pump. (Adapted from Ref. 10, used with permission.)**

12.6 Axial-Flow and Mixed-Flow Pumps

As noted previously, centrifugal pumps are radial-flow machines that operate most efficiently for applications requiring high heads at relatively low flowrates. This head–flowrate combination typically yields specific speeds (N_s) that are less than approximately 1.5. For many applications, such as those associated with drainage and irrigation, high flowrates at low heads are required and centrifugal pumps are not suitable. In this case, axial-flow pumps are commonly used. This type of pump consists essentially of a propeller confined within a cylindrical casing. Axial-flow pumps are often called *propeller pumps*. For this type of pump the flow is primarily in the axial direction (parallel to the axis of rotation of the shaft), as opposed to the radial flow found in the centrifugal pump. Whereas the head developed by a centrifugal pump includes a contribution due to centrifugal action, the head developed by an axial-flow pump is due primarily to the tangential force exerted by the rotor blades on the fluid. A schematic of an axial-flow pump arranged for vertical operation is shown in Fig. 12.19. The rotor is connected to a motor through a shaft, and as it rotates (usually at a relatively high speed) the fluid is sucked in through the inlet. Typically the fluid discharges through a row of fixed stator (guide) vanes used to straighten the flow leaving the rotor. Some axial-flow pumps also have inlet guide vanes upstream of the rotor row, and some are multistage in which pairs (*stages*) of rotating blades (*rotor blades*) and fixed vanes (*stator blades*) are arranged in series. Axial-flow pumps usually have specific speeds (N_s) in excess of 3.3.

Axial-flow pumps often have alternating rows of stator blades and rotor blades.

The definitions and broad concepts that were developed for centrifugal pumps are also applicable to axial-flow pumps. The actual flow characteristics, however, are quite different. In Fig. 12.20 typical head, power, and efficiency characteristics are compared for a centrifugal pump and an axial-flow pump. It is noted that at design capacity (maximum efficiency) the head and power are the same for the two pumps selected. But as the flowrate decreases, the power input to the centrifugal pump falls to 134 kW at shutoff, whereas for the axial-flow pump the power input increases to 388 kW at shutoff. This characteristic of the axial-flow pump can cause overloading of the drive motor if the flowrate is reduced significantly from the design capacity. It is also noted that the head curve for the axial-flow pump is much steeper than that for the centrifugal pump. Thus, with axial-flow pumps there will be a large change in head with a small change in the flowrate, whereas for the centrifugal pump, with its relatively flat head curve, there will be only a small change in head with large changes in the flowrate. It is further observed from Fig. 12.20 that, except at design capacity, the efficiency of the axial-flow pump is lower than that of the centrifugal pump. To improve operating characteristics, some axial-flow pumps are constructed with adjustable blades.

For applications requiring specific speeds intermediate to those for centrifugal and axial-flow pumps, mixed-flow pumps have been developed that operate efficiently in the specific speed range $1.5 < N_s < 3.3$. As the name implies, the flow in a mixed-flow pump has both a radial and an axial component. Figure 12.21 shows some typical data for centrifugal, mixed-flow, and axial-flow pumps, each operating with the same flowrate. These data indicate that as we proceed from the centrifugal pump to the mixed-flow pump to the axial-flow pump, the specific speed increases, the

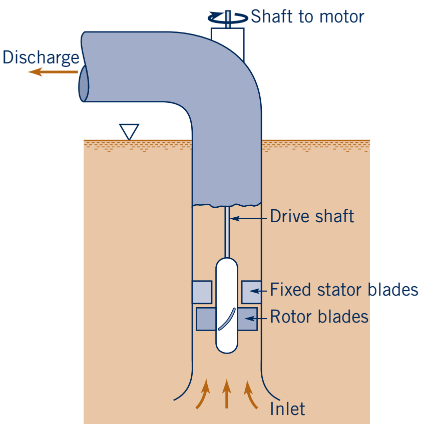
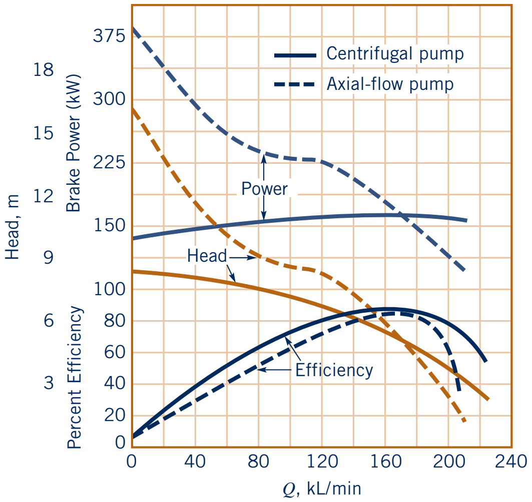


Figure 12.19 Schematic diagram of an axial-flow pump arranged for vertical operation.



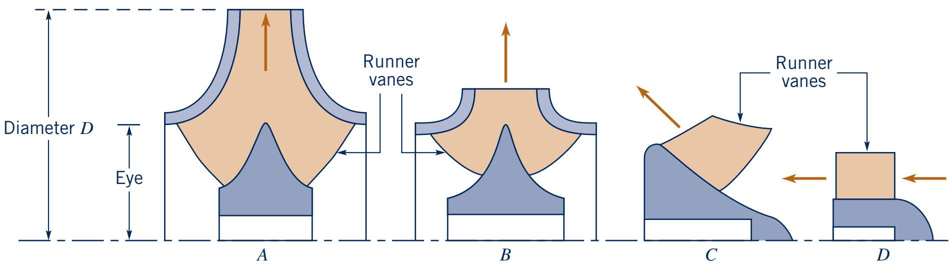
■ **Figure 12.20** **Comparison of performance characteristics for a centrifugal pump and an axial-flow pump, each rated 159 kW/min at a 5 m head. (Data from Ref. 12, used with permission.)**

head decreases, the speed increases, the impeller diameter decreases, and the eye diameter increases. These general trends are commonly found when these three types of pumps are compared.

The dimensionless parameters and scaling relationships developed in the previous sections apply to all three types of pumps—centrifugal, mixed-flow, and axial-flow—since the dimensional analysis used is not restricted to a particular type of pump. Additional information about pumps can be found in Refs. 4, 7, 9, 12, and 13.

Fluids in the News

Mechanical heart assist devices As with any pump, the human heart can suffer various malfunctions and problems during its useful life. Recent developments in artificial heart technology may be able to provide help to those whose pumps have broken down beyond repair. One of the more promising techniques is use of a left-ventricular assist device (LVAD), which supplements a diseased heart. Rather than replacing a diseased heart, an LVAD pump is implanted alongside the heart and works in parallel with the cardiovascular system to assist the pumping function of the heart's left ventricle. (The left ventricle supplies oxygenated blood to the entire body and performs about 80% of the heart's work.) Some LVADs are *centrifugal or axial-flow pumps* that provide a continuous flow of blood. The continuous-flow devices may take some adjustment on the part of patients who do not hear a pulse or a heartbeat. Despite advances in artificial heart technology, it is probably still several years before fully implantable, quiet, and reliable devices will be considered for widespread use.



Type	Centrifugal	Centrifugal	Mixed flow	Axial flow
N_s	0.5	0.8	2.3	5
kL/min	9	9	9	9
Head, km	21	15	10	6
Rpm	870	1160	1750	2600
D, cm	48	30	25	18
D_{eye}/D	0.5	0.7	0.9	1.0

■ **Figure 12.21** **Comparison of different types of impellers. Specific speed for centrifugal pumps based on single suction and identical flowrate. (Adapted from Ref. 12, used with permission.)**

12.7 Fans

Fans are used to pump air and other gases and vapors.

When the fluid to be moved is air, or some other gas or vapor, *fans* are commonly used. Types of fans vary from the small fan used for cooling desktop computers to large fans used in many industrial applications such as ventilating of large buildings. Fans typically operate at relatively low rotation speeds and are capable of moving large volumes of gas. Although the fluid of interest is a gas, the change in gas density through the fan does not usually exceed 7%, which for air represents a change in pressure of only about 7 kPa (Ref. 14). Thus, in dealing with fans, the gas density is treated as a constant, and the flow analysis is based on incompressible flow concepts. Because of the low-pressure rise involved, fans are often constructed of lightweight sheet metal. Fans are also called *blowers*, *boosters*, and *exhausters,* depending on the location within the system; that is, blowers are located at the system entrance, exhausters are at the system exit, and boosters are at some intermediate position within the system. Turbomachines used to produce larger changes in gas density and pressure than is possible with fans are called *compressors* (see Section 12.9.1).

As is the case for pumps, fan designs include centrifugal (radial-flow) fans, as well as mixed-flow and axial-flow (propeller) fans, and the analysis of fan performance closely follows that previously described for pumps. The shapes of typical performance curves for centrifugal and axial-flow fans are quite similar to those shown in Fig. 12.20 for centrifugal and axial-flow pumps. However, fan head-rise data are often given in terms of pressure rise, either static or total, rather than the more conventional head rise commonly used for pumps.

Scaling relationships for fans are the same as those developed for pumps; that is, Eqs. 12.32 through 12.35 apply to fans as well as pumps. As noted above, for fans it is common to replace the head, h_a, in Eq. 12.33 with pressure head, $p_a/\rho g$, so that Eq. 12.33 becomes

$$\left(\frac{p_a}{\rho\omega^2 D^2}\right)_1 = \left(\frac{p_a}{\rho\omega^2 D^2}\right)_2 \quad \textbf{(12.47)}$$

where, as before, the subscripts 1 and 2 refer to any two fans from the family of geometrically similar fans. Equations 12.47, 12.32, and 12.34 are called the *fan laws* and can be used to scale performance characteristics between members of a family of geometrically similar fans. Additional information about fans can be found in Refs. 14–17.

Fluids in the News

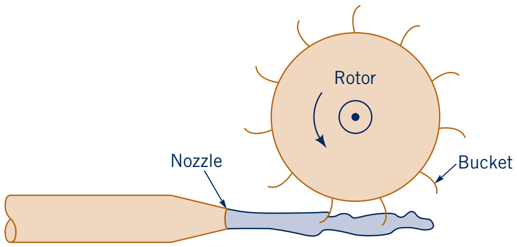
Hi-tech ceiling fans Energy savings of up to 25% can be realized if thermostats in air-conditioned homes are raised by a few degrees. This can be accomplished by using ceiling *fans* and taking advantage of the increased sensible cooling brought on by air moving over skin. If the energy used to run the fans can be reduced, additional energy savings can be realized. Most ceiling fans use flat, fixed pitch, nonaerodynamic *blades* with uniform chord length. Because the tip of a paddle moves through air faster than its root does, airflow over such fan blades is lowest near the hub and highest at the tip. By making the fan blade more propeller-like, it is possible to have a more uniform, efficient distribution. However, since ceiling fans are restricted by law to operate at less than 200 rpm, ordinary airplane propeller design is not appropriate. After considerable design effort, a highly efficient ceiling fan capable of delivering the same airflow as the conventional design with only half the power has been successfully developed and marketed. The fan blades are based on the slowly turning prop used in the *Gossamer Albatross*, the human-powered aircraft that flew across the English Channel in 1979. (See Problem 12.46.)

12.8 Turbines

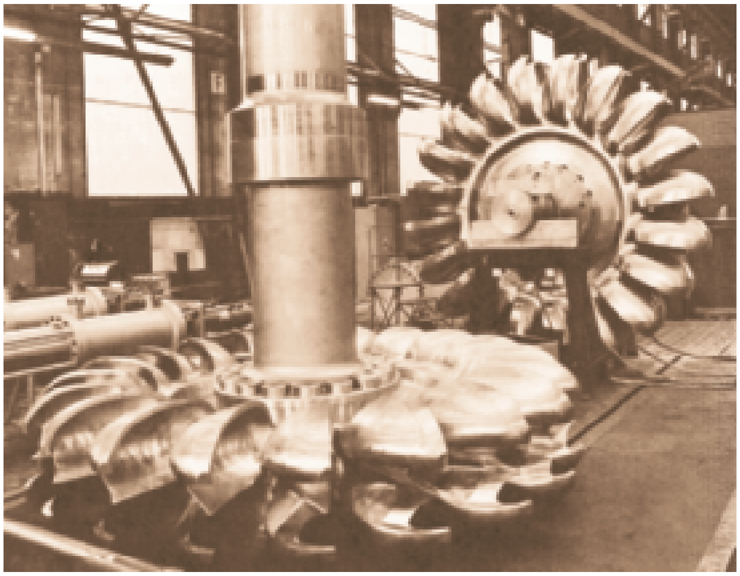
As discussed in Section 12.2, turbines are devices that extract energy from a flowing fluid. The geometry of turbines is such that the fluid exerts a torque on the rotor in the direction of its rotation. The shaft power generated is available to drive generators or other devices.

In the following sections we discuss mainly the operation of hydraulic turbines (those for which the working fluid is water) and to a lesser extent gas and steam turbines (those for which the density of the working fluid may be much different at the inlet than at the outlet).

Although there are numerous ingenious hydraulic turbine designs, most of these turbines can be classified into two basic types—***impulse turbines*** and ***reaction turbines***. (Reaction is related to



(*a*)



(*b*)

■ **Figure 12.22** ***(a)* Schematic diagram of a Pelton wheel turbine, *(b)* photograph of a Pelton wheel turbine. (Courtesy of Voith Hydro, York, PA.)**

The two basic types of hydraulic turbines are impulse and reaction.

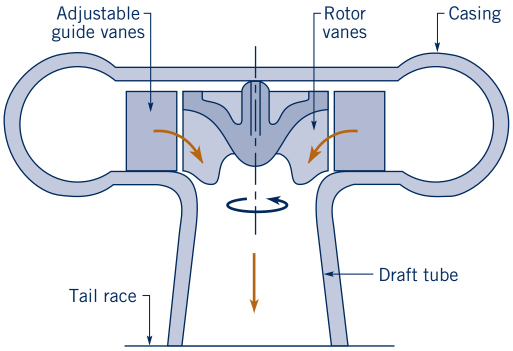
the ratio of static pressure drop that occurs across the rotor to static pressure drop across the turbine stage, with larger rotor pressure drop corresponding to larger reaction.) For hydraulic impulse turbines, the pressure drop across the rotor is zero; all of the pressure drop across the turbine stage occurs in the nozzle row. The ***Pelton wheel*** shown in Fig. 12.22 is a classical example of an impulse turbine. In these machines the total head of the incoming fluid (the sum of the pressure head, velocity head, and elevation head) is converted into a large-velocity head at the exit of the supply nozzle (or nozzles if a multiple nozzle configuration is used). Both the pressure drop across the bucket (blade) and the change in relative speed (i.e., fluid speed relative to the moving bucket) of the fluid across the bucket are negligible. The space surrounding the rotor is not completely filled with fluid. It is the impulse of the individual jets of fluid striking the buckets that generates the torque.

For reaction turbines, on the other hand, the rotor is surrounded by a casing (or volute), which is completely filled with the working fluid. There is both a pressure drop and a fluid relative speed change across the rotor. As shown for the radial-inflow turbine in Fig 12.23, guide vanes act as nozzles to accelerate the flow and turn it in the appropriate direction as the fluid enters the rotor. Thus, part of the pressure drop occurs across the guide vanes and part occurs across the rotor. In many respects the operation of a reaction turbine is similar to that of a pump "flowing backward," although such oversimplification can be quite misleading.

Both impulse and reaction turbines can be analyzed using the moment-of-momentum principles discussed in Section 12.3. In general, impulse turbines are high-head, low-flowrate devices, while reaction turbines are low-head, high-flowrate devices.

12.8.1 Impulse Turbines

Although there are various types of impulse turbine designs, perhaps the easiest to understand is the Pelton wheel (see Fig. 12.24). Lester Pelton (1829–1908), an American mining engineer during the



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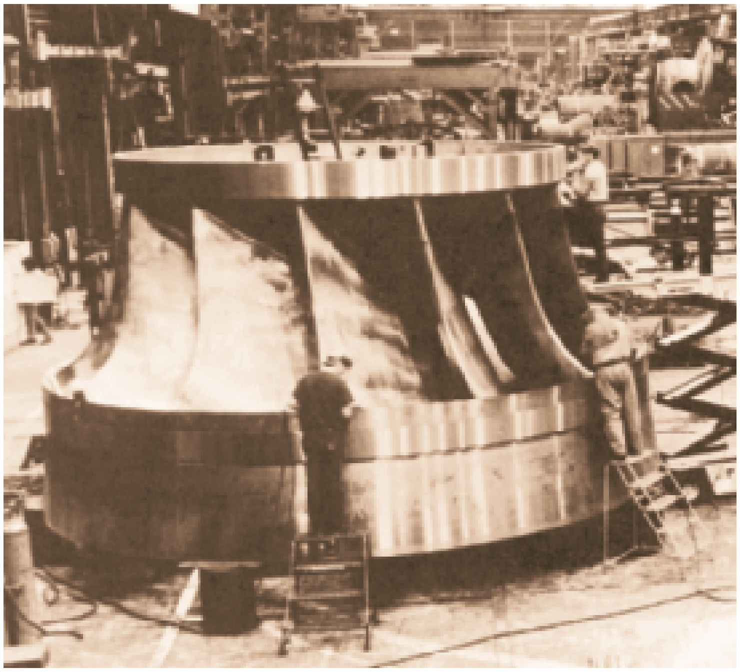
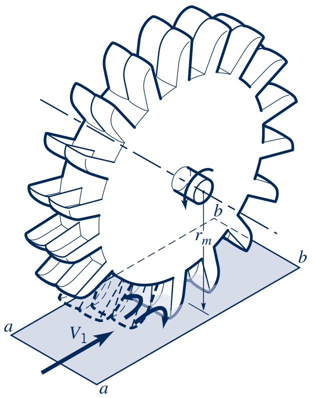


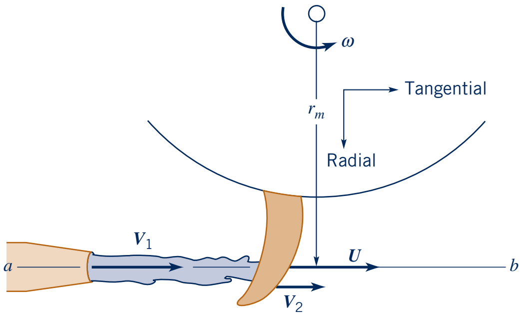
(*b*)

■ **Figure 12.23** **(*a*) Schematic diagram of a reaction turbine, (*b*) photograph of a reaction turbine. (Courtesy of Voith Hydro, York, PA.)**

California gold-mining days, is responsible for many of the still-used features of this type of turbine. It is most efficient when operated with a large head (for example, a water source from a lake located significantly above the turbine nozzle), which is converted into a relatively large velocity at the exit of the nozzle. Among the many design considerations for such a turbine are the head loss that occurs in the pipe (the penstock) transporting the water to the turbine, the design of the nozzle, and the design of the buckets on the rotor.



■ **Figure 12.24** **Details of Pelton wheel turbine bucket.**



■ **Figure 12.25** **Ideal fluid velocities for a Pelton wheel turbine.**

Pelton wheel turbines operate most efficiently with a larger head and lower flowrates.

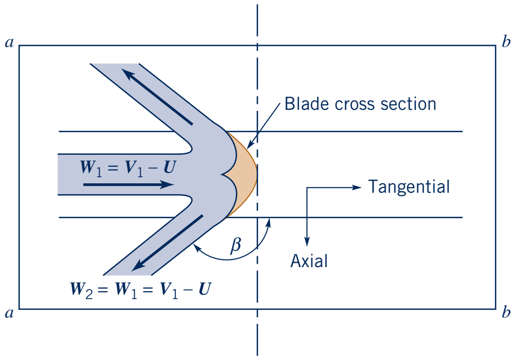
V12.4 Pelton wheel lawn sprinkler

As shown in Fig. 12.24, a high-speed jet of water strikes the Pelton wheel buckets and is deflected. The water enters and leaves the control volume surrounding the wheel as free jets (atmospheric pressure). In addition, a person riding on the bucket would note that the speed of the water does not change as it slides across the buckets (assuming viscous effects are negligible). That is, the magnitude of the relative velocity does not change, but its direction does. The change in direction of the velocity of the fluid jet causes a torque on the rotor, resulting in a power output from the turbine.

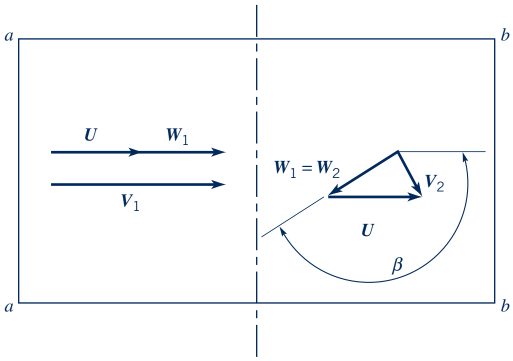
Design of the optimum, complex shape of the buckets to obtain maximum power output is a very difficult matter. Ideally, the fluid enters and leaves the control volume shown in Fig. 12.25 with no radial component of velocity. (In practice there often is a small but negligible radial component.) In addition, the buckets would ideally turn the relative velocity vector through a 180° turn, but physical constraints dictate that β, the angle of the exit edge of the blade, is less than 180°. Thus, the fluid leaves with an axial component of velocity as shown in Fig. 12.26.

The inlet and exit velocity triangles at the arithmetic mean radius, r_m, are assumed to be as shown in Fig. 12.27. To calculate the torque and power, we must know the tangential components of the absolute velocities at the inlet and exit. (Recall from the discussion in Section 12.3 that neither the radial nor the axial components of velocity enter into the torque or power equations.) From Fig. 12.27 we see that

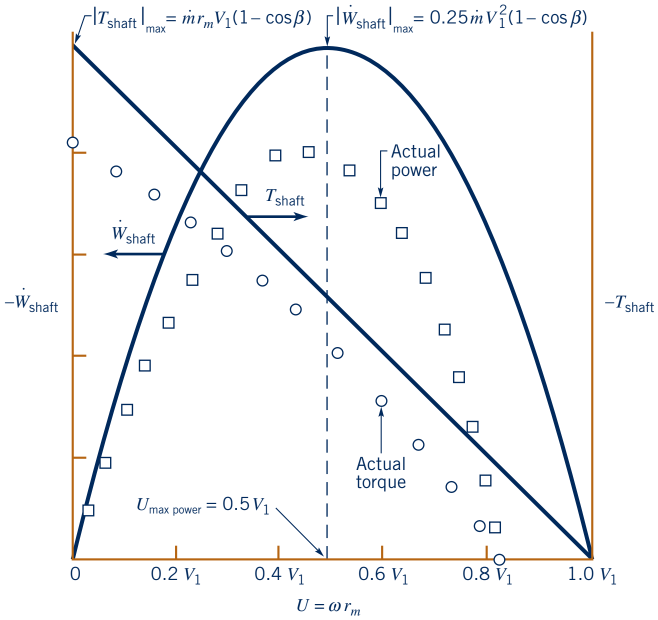
$$V_{\theta 1} = V_1 = W_1 + U \tag{12.48}$$



■ **Figure 12.26** **Flow as viewed by an observer riding on the Pelton wheel—relative velocities.**



■ **Figure 12.27** **Inlet and exit velocity triangles for a Pelton wheel turbine.**



■ **Figure 12.28** **Typical theoretical and experimental power and torque for a Pelton wheel turbine as a function of bucket speed.**

and

$$V_{\theta 2} = W_2 \cos\beta + U \tag{12.49}$$

In Pelton wheel analyses, we assume the relative speed of the fluid is constant (no friction).

Thus, with the assumption that $W_1 = W_2$ (i.e., the relative speed of the fluid does not change as it is deflected by the buckets), we can combine Eqs. 12.48 and 12.49 to obtain

$$V_{\theta 2} - V_{\theta 1} = (U - V_1)(1 - \cos\beta) \tag{12.50}$$

This change in tangential component of velocity combined with the torque and power equations developed in Section 12.3 (i.e., Eqs. 12.2 and 12.4) gives

$$T_{shaft} = \dot{m} r_m (U - V_1)(1 - \cos\beta)$$

where $\dot{m} = \rho Q$ is the mass flowrate through the turbine. Since $U = \omega R_m$, it follows that

$$\dot{W}_{shaft} = T_{shaft}\omega = \dot{m} U(U - V_1)(1 - \cos\beta) \tag{12.51}$$

These results are plotted in Fig. 12.28 along with typical experimental results. Note that $V_1 > U$ (i.e., the jet impacts the bucket), and $\dot{W}_{shaft} < 0$ (i.e., the turbine extracts power from the fluid).

Several interesting points can be noted from the above results. First, the power is a function of β. However, a typical value of $\beta = 165°$ (rather than the optimum 180°) results in a relatively small (less than 2%) reduction in power since $1 - \cos 165° = 1.966$, compared to $1 - \cos 180° = 2$. Second, although the torque is maximum when the wheel is stopped ($U = 0$), there is no power under this condition—to extract power one needs force and motion. On the other hand, the power output is a maximum when

$$U\big|_{max\ power} = \frac{V_1}{2} \tag{12.52}$$

This can be shown by using Eq. 12.51 and solving for U that gives $d\dot{W}_{shaft}/dU = 0$. A bucket speed of one-half the speed of the fluid coming from the nozzle gives the maximum power. Third, the maximum speed occurs when $T_{shaft} = 0$ (i.e., the load is completely removed from the turbine, as would happen if the shaft connecting the turbine to the generator were to break and frictional torques were negligible). For this case $U = \omega R = V_1$, the turbine is "freewheeling," and the water simply passes across the rotor without putting any force on the buckets.

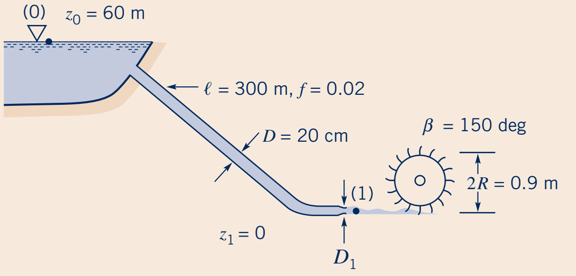
Although the actual flow through a Pelton wheel is considerably more complex than assumed in the above simplified analysis, reasonable results and trends are obtained by this simple application of the moment-of-momentum principle.

EXAMPLE 12.6 Pelton Wheel Turbine Characteristics

GIVEN Water to drive a Pelton wheel is supplied through a pipe from a lake as indicated in Fig. E12.6*a*. The head loss due to friction in the pipe is important, but minor losses can be neglected.

FIND **(a)** Determine the nozzle diameter, D_1, that will give the maximum power output.

(b) Determine the maximum power and the angular velocity of the rotor at the conditions found in part **(a)**.



■ **Figure E12.6*a***

SOLUTION

(a) As indicated by Eq. 12.51, the power output depends on the flowrate, $Q = \dot{m}/\rho$, and the jet speed at the nozzle exit, V_1, both of which depend on the diameter of the nozzle, D_1, and the head loss associated with the supply pipe. That is

$$\dot{W}_{\text{shaft}} = \rho Q U (U - V_1)(1 - \cos\beta) \tag{1}$$

The nozzle exit speed, V_1, can be obtained by applying the energy equation (Eq. 5.85) between a point on the lake surface (where $V_0 = p_0 = 0$) and the nozzle outlet (where $z_1 = p_1 = 0$) to give

$$z_0 = \frac{V_1^2}{2g} + h_L \tag{2}$$

where the head loss is given in terms of the friction factor, *f*, as (see Eq. 8.34)

$$h_L = f\frac{\ell}{D}\frac{V^2}{2g}$$

The speed, *V*, of the fluid in the pipe of diameter *D* is obtained from the continuity equation

$$V = \frac{A_1 V_1}{A} = \left(\frac{D_1}{D}\right)^2 V_1$$

We have neglected minor losses associated with the pipe entrance and the nozzle. With the given data, Eq. 2 becomes

$$z_0 = \left[1 + f\frac{\ell}{D}\left(\frac{D_1}{D}\right)^4\right]\frac{V_1^2}{2g} \tag{3}$$

or

$$V_1 = \left[\frac{2gz_0}{1 + f\dfrac{\ell}{D}\left(\dfrac{D_1}{D}\right)^4}\right]^{1/2}$$

$$= \left[\frac{2(9.8\text{ m/s}^2)(60\text{ m})}{1 + 0.02\left(\dfrac{300\text{ m}}{20/100\text{ m}}\right)\left(\dfrac{D_1}{20/100}\right)^4}\right]^{1/2}$$

$$= \frac{34.3}{\sqrt{1 + 18{,}750\, D_1^4}} \tag{4}$$

where D_1 is in meters.

By combining Eqs. 1 and 4 and using $Q = \pi D_1^2 V_1/4$ we obtain the power as a function of D_1 and U as

$$\dot{W}_{\text{shaft}} = \frac{5.02 \times 10^4\, U D_1^2}{\sqrt{1 + 18{,}750\, D_1^4}}\left[U - \frac{113.5}{\sqrt{1 + 152\, D_1^4}}\right] \tag{5}$$

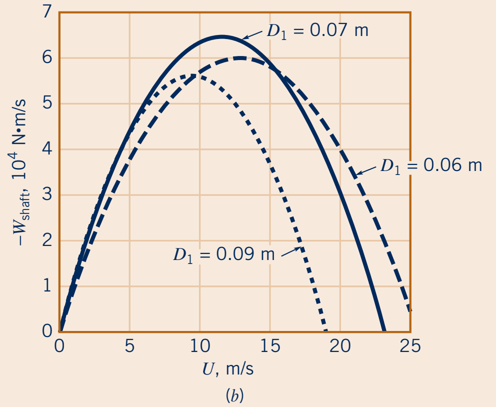
where U is in meters per second and $\dot{W}_{\text{shaft}}$ is in N · m/s. These results are plotted as a function of U for various values of D_1 in Fig. E12.6*b*.

As shown by Eq. 12.52, the maximum power (in terms of its variation with U) occurs when $U = V_1/2$, which, when used with Eqs. 4 and 5, gives

$$\dot{W}_{\text{shaft}} = -\frac{15 \times 10^6\, D_1^2}{(1 + 18{,}750\, D_1^4)^{3/2}} \tag{6}$$

The maximum power possible occurs when $d\dot{W}_{\text{shaft}}/dD_1 = 0$, which according to Eq. 6 can be found as

$$\frac{d\dot{W}_{\text{shaft}}}{dD_1} = -15 \times 10^6\left[\frac{2\,D_1}{(1 + 18{,}750\, D_1^4)^{3/2}} - \left(\frac{3}{2}\right)\frac{4(18{,}750)\, D_1^5}{(1 + 18{,}750\, D_1^4)^{5/2}}\right] = 0$$



■ **Figure E12.6*b***

or

$$37{,}250\, D_1^4 = 1$$

Thus, the nozzle diameter for maximum power output is

$$D_1 = 0.07 \text{ m} \qquad \textbf{(Ans)}$$

(b) The corresponding maximum power can be determined from Eq. 6 as

$$\dot{W}_{\text{shaft}} = -\frac{15 \times 10^6\,(0.07)^2}{[1 + 18{,}750(0.07)^4]^{3/2}} = -4.2 \times 10^4 \text{ N} \cdot \text{m/s}$$

or

$$\dot{W}_{\text{shaft}} = -4.2 \times 10^4 \text{ N} \cdot \text{m} \qquad \textbf{(Ans)}$$

The rotor speed at the maximum power condition can be obtained from

$$U = \omega R = \frac{V_1}{2}$$

where V_1 is given by Eq. 4. Thus,

$$\omega = \frac{V_1}{2R} = \frac{\dfrac{34.3}{\sqrt{1 + 18{,}750(0.07)^4}}\text{ m/s}}{2(0.9 \text{ m})}$$
$$= 31.6 \times 1 \text{ rev}/2\pi \text{ rad} \times 60 \text{ s/min}$$
$$= 302 \text{ rpm} \qquad \textbf{(Ans)}$$

COMMENT The reason that an optimum diameter nozzle exists can be explained as follows. A larger-diameter nozzle will allow a larger flowrate but will produce a smaller jet velocity because of the head loss within the supply side. A smaller-diameter nozzle will reduce the flowrate but will produce a larger jet velocity. Since the power depends on a product combination of flowrate and jet velocity (see Eq. 1), there is an optimum-diameter nozzle that gives the maximum power.

These results can be generalized (i.e., without regard to the specific parameter values of this problem) by considering Eqs. 1 and 3 and the condition that $U = V_1/2$ to obtain

$$\dot{W}_{\text{shaft}\,|U=V_1/2} = -\frac{\pi}{16}\rho(1 - \cos\beta) \times (2gz_0)^{3/2} D_1^2 \Big/ \left(1 + f\frac{\ell}{D^5}D_1^4\right)^{3/2}$$

By setting $d\dot{W}_{\text{shaft}}/dD_1 = 0$, it can be shown (see Problem 12.61) that the maximum power occurs when

$$D_1 = D\Big/\left(2f\frac{\ell}{D}\right)^{1/4}$$

which gives the same results obtained earlier for the specific parameters of the example problem. Note that the optimum condition depends only on the friction factor and the length-to-diameter ratio of the supply pipe. What happens if the supply pipe is frictionless or of essentially zero length?

In previous chapters, we mainly treated turbines (and pumps) as "black boxes" in the flow that removed (or added) energy to the fluid. We treated these devices as objects that removed a certain shaft work head from or added a certain shaft work head to the fluid. The relationship between the shaft work head and the power output as described by the moment-of-momentum considerations is illustrated in Example 12.7.

EXAMPLE 12.7 Maximum Power Output for a Pelton Wheel Turbine

GIVEN Water flows through the Pelton wheel turbine shown in Fig. 12.24. For simplicity we assume that the water is turned 180° by the blade.

FIND Show, based on the energy equation (Eq. 5.84), that the maximum power output occurs when the absolute velocity of the fluid exiting the turbine is zero.

SOLUTION

As indicated by Eq. 12.51, the shaft power of the turbine is given by

$$\dot{W}_{\text{shaft}} = \rho QU(U - V_1)(1 - \cos\beta)$$
$$= 2\rho Q(U^2 - V_1U) \qquad \textbf{(1)}$$

For this impulse turbine with $\beta = 180°$, the velocity triangles simplify into the diagram types shown in Fig. E12.7. Three possibilities are indicated:

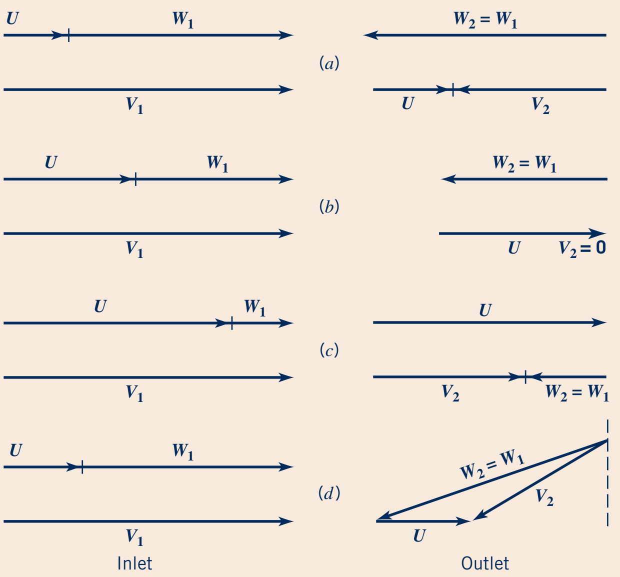
(a) the exit absolute velocity, $\mathbf{V}_2$, is directed back toward the nozzle,

(b) the absolute velocity at the exit is zero, or

(c) the exiting stream flows in the direction of the incoming stream.

According to Eq. 12.52, the maximum power occurs when $U = V_1/2$, which corresponds to the situation shown in Fig. E12.7*b*, that is, $U = V_1/2 = W_1$. If viscous effects are negligible, then $W_1 = W_2$ and we have $U = W_2$, which gives

$$V_2 = 0 \qquad \textbf{(Ans)}$$



■ **Figure E12.7**

If we consider the energy equation (Eq. 5.84) for flow across the rotor, we have

$$\frac{p_1}{\gamma} + \frac{V_1^2}{2g} + z_1 + h_S = \frac{p_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_L$$

where h_S is the shaft work head. This simplifies to

$$h_S = \frac{V_2^2 - V_1^2}{2g} + h_L \qquad \textbf{(2)}$$

since $p_1 = p_2$ and $z_1 = z_2$. Note that the impulse turbine obtains its energy from a reduction in the velocity head. The largest shaft work head possible (and therefore the largest power) occurs when all of the kinetic energy available is extracted by the turbine, giving

$$V_2 = 0 \qquad \textbf{(Ans)}$$

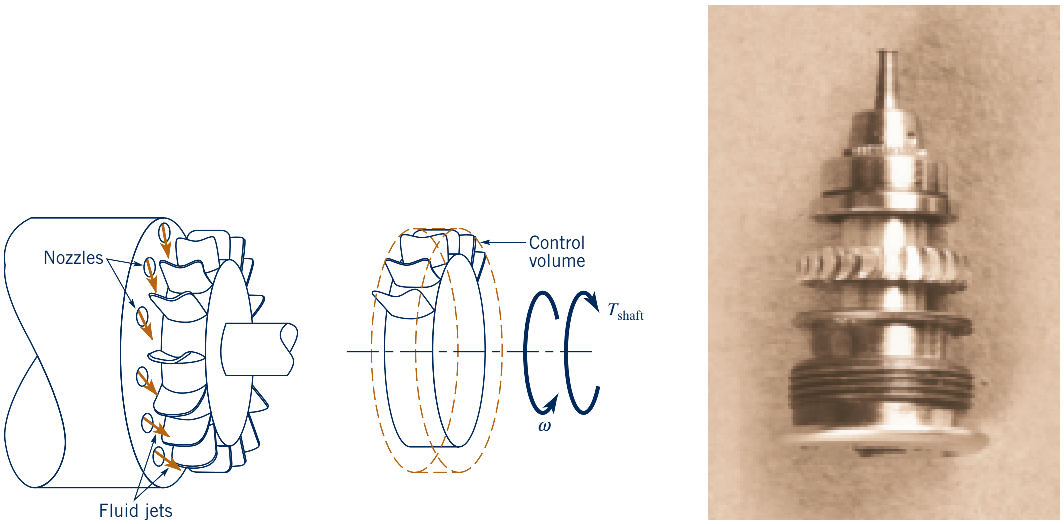
This is consistent with the maximum power condition represented by Fig. E12.7*b*.

COMMENT As indicated by Eq. 1, if the exit absolute velocity is not in the plane of the rotor (i.e., $\beta < 180°$), there is a reduction in the power available (by a factor of $1 - \cos\beta$). This is also supported by the energy equation, Eq. 2, as follows. For $\beta < 180°$ the inlet and exit velocity triangles are as shown in Fig. E12.7*d*. Regardless of the bucket speed, U, it is not possible to reduce the value of V_2 to zero—there is always a component in the axial direction. Thus, according to Eq. 2, the turbine cannot extract the entire velocity head; the exiting fluid has some kinetic energy left in it.

A second type of impulse turbine that is widely used (most often with air as the working fluid) is indicated in Fig. 12.29. A circumferential series of fluid jets strikes the rotating blades which, as with the Pelton wheel, alter both the direction and magnitude of the absolute velocity. As with the Pelton wheel, the inlet and exit pressures (i.e., on either side of the rotor) are equal, and the magnitude of the relative velocity is unchanged as the fluid slides across the blades (if frictional effects are negligible).

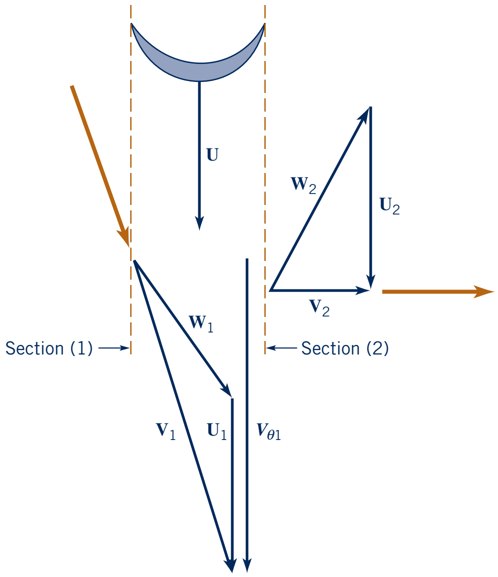
Dentist drill turbines are usually of the impulse class.

Typical inlet and exit velocity triangles (absolute, relative, and blade velocities) are shown in Fig. 12.30. As discussed in Section 12.2, in order for the absolute velocity of the fluid to be changed



(*a*) (*b*)

■ **Figure 12.29** **(*a*) A multinozzle, non-Pelton wheel impulse turbine commonly used with air as the working fluid, (*b*) dental drill.**



■ **Figure 12.30** **Inlet and exit velocity triangles for the impulse turbine shown in Fig. 12.29.**

as indicated during its passage across the blade, the blade must push on the fluid in the direction opposite of the blade motion. Hence, the fluid pushes on the blade in the direction of the blade's motion—the fluid does work on the blade (a turbine).

EXAMPLE 12.8 Non-Pelton Wheel Impulse Turbine (Dental Drill)

GIVEN An air turbine used to drive the high-speed drill used by your dentist is shown in Figs. 12.29 and E12.8*a* (also see **Video V12.5**). Air exiting from the upstream nozzle holes forces the turbine blades to move in the direction shown. The turbine rotor speed is 300,000 rpm, the tangential component of velocity out of the nozzle is twice the blade speed, and the tangential component of the absolute velocity out of the rotor is zero.

FIND Estimate the shaft energy per unit mass of air flowing through the turbine.

SOLUTION

We use the fixed, nondeforming control volume that includes the turbine rotor and the fluid in the rotor blade passages at an instant of time (see Fig. E12.8*b*). The only torque acting on this control volume is the shaft torque. For simplicity we analyze this problem using an arithmetic mean radius, r_m, where

$$r_m = \frac{1}{2}(r_0 + r_i)$$

A sketch of the velocity triangles at the rotor entrance and exit is shown in Fig. E12.8*c*.

Application of Eq. 12.5 (a form of the moment-of-momentum equation) gives

$$w_{\text{shaft}} = -U_1 V_{\theta 1} + U_2 V_{\theta 2} \quad \textbf{(1)}$$

where w_{shaft} is shaft energy per unit of mass flowing through the turbine. From the problem statement, $V_{\theta 1} = 2U$ and $V_{\theta 2} = 0$, where

$$\begin{aligned} U = \omega r_m &= (300{,}000 \text{ rev/min})(1 \text{ min}/60 \text{ s})(2\pi \text{ rad/rev}) \\ &\quad \times (0.43 \text{ cm} + 0.34 \text{ cm})/2(100 \text{ cm/m}) \quad \textbf{(2)} \\ &= 121 \text{ m/s} \end{aligned}$$

is the mean-radius blade velocity. Thus, Eq. (1) becomes

$$\begin{aligned} w_{\text{shaft}} &= -U_1 V_{\theta 1} = -2U^2 = -2(121 \text{ m/s})^2 \\ &= -29.3 \times 10^3 \text{ m}^2/\text{s}^2 \\ &= (-29.3 \times 10^3 \text{ m}^2/\text{s}^2)/(\text{m} \cdot \text{kg/N} \cdot \text{s}^2) \\ &= -29 \text{ kN} \cdot \text{m/kg} \end{aligned} \quad \textbf{(Ans)}$$

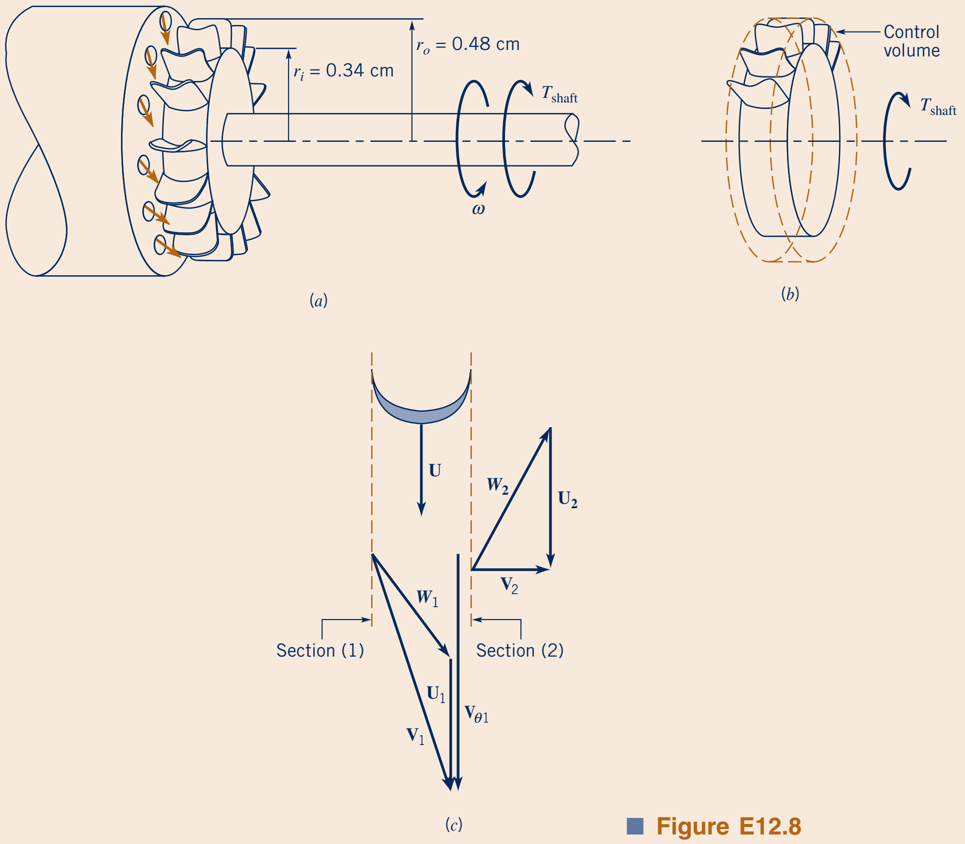
COMMENT For each kg of air passing through the turbine there is 29 kN · m/kg of energy available at the shaft to drive the drill. However, because of fluid friction, the actual amount of energy given up by each gram of air will be greater than the amount available at the shaft. How much greater depends on the efficiency of the fluid-mechanical energy transfer between the fluid and the turbine blades.

Recall that the shaft power, $\dot{W}_{\text{shaft}}$, is given by

$$\dot{W}_{\text{shaft}} = \dot{m} w_{\text{shaft}}$$

Hence, to determine the power, we need to know the mass flowrate, $\dot{m}$, which depends on the size and number of the nozzles. Although the energy per unit mass is large (i.e., 29 kN · m/kg), the flowrate is small, so the power is not "large."

V12.5 Dental drill



■ **Figure E12.8**

12.8.2 Reaction Turbines

Reaction turbines are best suited for higher flowrate and lower head situations.

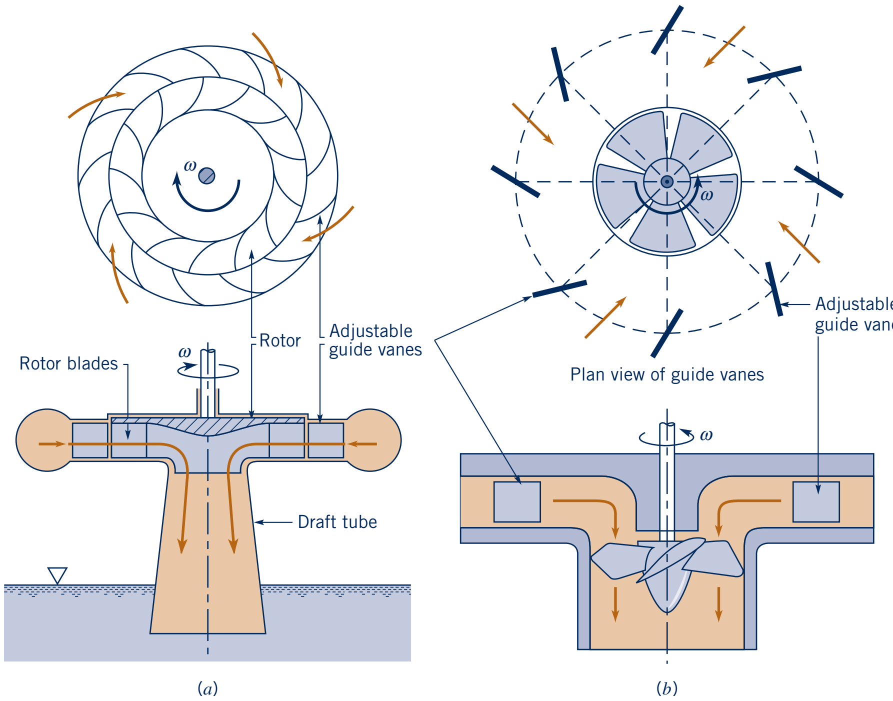
As indicated in the previous section, impulse turbines are best suited (i.e., most efficient) for lower-flowrate and higher-head operations. Reaction turbines, on the other hand, are best suited for higher-flowrate and lower-head situations such as are often encountered in hydroelectric power plants associated with a dammed river, for example.

In a reaction turbine the working fluid completely fills the passageways through which it flows (unlike an impulse turbine, which contains one or more individual unconfined jets of fluid). The angular momentum, pressure, and velocity of the fluid decrease as it flows through the turbine rotor—the turbine rotor extracts energy from the fluid.

As with pumps, turbines are manufactured in a variety of configurations—radial-flow, mixed-flow, and axial-flow types. Typical radial- and mixed-flow hydraulic turbines are called *Francis turbines*, named after James B. Francis, an American engineer. At very low heads the most efficient type of turbine is the axial-flow or propeller turbine. The *Kaplan turbine*, named after Victor Kaplan, a German professor, is an efficient axial-flow hydraulic turbine with adjustable blades. Cross sections of these different turbine types are shown in Fig. 12.31.

As shown in Fig. 12.31*a*, flow across the rotor blades of a radial-inflow turbine has a major component in the radial direction. Inlet guide vanes (which may be adjusted to allow optimum performance) direct the water into the rotor with a tangential component of velocity. The absolute velocity of the water leaving the rotor is essentially without tangential velocity. Hence, the rotor decreases the angular momentum of the fluid, the fluid exerts a torque on the rotor in the direction of rotation, and the rotor extracts energy from the fluid. The Euler turbomachine equation (Eq. 12.2) and the corresponding power equation (Eq. 12.4) are equally valid for this turbine as they are for the centrifugal pump discussed in Section 12.4.

As shown in Fig. 12.31*b*, for an axial-flow Kaplan turbine, the fluid flows through the inlet guide vanes and achieves a tangential velocity in a vortex (swirl) motion before it reaches the



■ **Figure 12.31** ***(a)* Typical radial-flow Francis turbine, *(b)* typical axial-flow Kaplan turbine.**

rotor. Flow across the rotor contains a major axial component. Both the inlet guide vanes and the turbine blades can be adjusted by changing their setting angles to produce the best match (optimum output) for the specific operating conditions. For example, the operating head available may change from season to season and/or the flowrate through the rotor may vary.

Fluids in the News

Fish friendly hydraulic turbine Based on data about what actually kills fish as they pass through *hydraulic turbines*, Concepts NREC produced a rotor design that allows a larger flow passage, a more uniform pressure distribution, lower levels of shear stress, and other acceptable trade-offs between efficiency and fish survivability. Tests and projections suggest that the fish friendly turbine design will achieve 90% efficiency, with fish survivability increased from 60 to 98%.

Pumps and turbines are often thought of as the "inverse" of each other. Pumps add energy to the fluid; turbines remove energy. The propeller on an outboard motor (a pump) and the propeller on a Kaplan turbine are in some ways geometrically similar, but they perform opposite tasks. Similar comparisons can be made for centrifugal pumps and Francis turbines. In fact, some large turbomachines at hydroelectric power plants are designed to be run as turbines during high-power demand periods (i.e., during the day) and as pumps to resupply the upstream reservoir from the downstream reservoir during low-demand times (i.e., at night). Thus, a pump type often has its corresponding turbine type. However, is it possible to have the "inverse" of a Pelton wheel turbine—an impulse pump?

Actual head available for a turbine, h_a, is always greater than shaft work head, h_s, because of head loss, h_L, in the turbine.

As with pumps, incompressible flow turbine performance is often specified in terms of appropriate dimensionless parameters. The flow coefficient, $C_Q = Q/\omega D^3$, the head coefficient, $C_H = gh_a/\omega^2 D^2$, and the power coefficient, $C_{\mathcal{P}} = \dot{W}_{\text{shaft}}/\rho\omega^3 D^5$, are defined in the same way for pumps and turbines. On the other hand, turbine efficiency, η, is the inverse of pump efficiency. That is, the efficiency is the ratio of the shaft power output to the power available in the flowing fluid, or

$$\eta = \frac{\dot{W}_{\text{shaft}}}{\rho g Q h_a}$$

For geometrically similar turbines and for negligible Reynolds number and surface roughness difference effects, the relationships between the dimensionless parameters are given functionally by that shown in Eqs. 12.29, 12.30, and 12.31. That is,

$$C_H = \phi_1(C_Q), \quad C_{\mathcal{P}} = \phi_2(C_Q), \quad \text{and} \quad \eta = \phi_3(C_Q)$$

where the functions ϕ_1, ϕ_2, and ϕ_3 are dependent on the type of turbine involved. Also, for turbines the efficiency, η, is related to the other coefficients according to $\eta = C_{\mathcal{P}}/C_H C_Q$.

As indicated above, the design engineer has a variety of turbine types available for any given application. It is necessary to determine which type of turbine would best fit the job (i.e., be most efficient) before detailed design work is attempted. As with pumps, the use of a specific speed parameter can help provide this information. For hydraulic turbines, the rotor diameter D is eliminated between the flow coefficient and the power coefficient to obtain the *power specific speed*, N_s', where

$$N_s' = \frac{\omega\sqrt{\dot{W}_{\text{shaft}}/\rho}}{(gh_{\text{a}})^{5/4}}$$

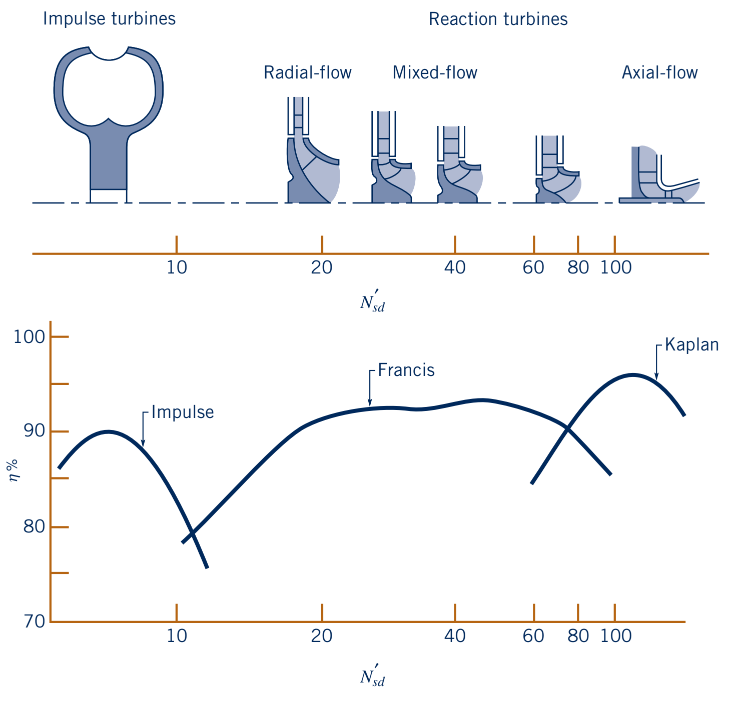
We use the more common, but not dimensionless, definition of specific speed

$$N_{sd}' = \frac{\omega(\text{rpm})\sqrt{\dot{W}_{\text{shaft}}\,(\text{W})}}{[h_{\text{a}}\,(\text{m})]^{5/4}} \quad \textbf{(12.53)}$$

Specific speed may be used to approximate what kind of turbine geometry (axial to radial) would operate most efficiently.

That is, N_{sd}' is calculated with angular velocity, ω, in rpm; shaft power, $\dot{W}_{\text{shaft}}$, in brake power (kW); and actual head available, h_{a}, in meters. Optimum turbine efficiency (for large turbines) as a function of specific speed is indicated in Fig. 12.32. Also shown are representative rotor and casing cross sections. Note that impulse turbines are best at low specific speeds; that is, when operating with large heads and small flowrate. The other extreme is axial-flow turbines, which are the most efficient type if the head is low and if the flowrate is large. For intermediate values of specific speeds, radial- and mixed-flow turbines offer the best performance.

The data shown in Fig. 12.32 are meant only to provide a guide for turbine-type selection. The actual turbine efficiency for a given turbine depends very strongly on the detailed design of the turbine. Considerable analysis, testing, and experience are needed to produce an efficient turbine. However, the data of Fig. 12.32 are representative. Much additional information can be found in the literature.



■ **Figure 12.32** **Typical turbine cross sections and maximum efficiencies as a function of specific speed.**

Fluids in the News

Cavitation damage in hydraulic turbines The occurrence of *cavitation* in hydraulic *pumps* seems to be an obvious possibility since low-suction *pressures* are expected. Cavitation damage can also occur in hydraulic *turbines,* even though they do not seem obviously prone to this kind of problem. Local *acceleration* of liquid over *blade* surfaces can be sufficient to result in local pressures low enough to cause fluid vaporization or cavitation. Further along the flow path, the fluid can decelerate rapidly enough with accompanying increase in local pressure to make cavitation bubbles collapse with enough intensity to cause blade surface damage in the form of material erosion. Over time, this erosion can be severe enough to require blade repair or replacement which is very expensive. (See Problem 12.4LL.)

EXAMPLE 12.9 Use of Specific Speed to Select Turbine Type

GIVEN A hydraulic turbine is to operate at an angular velocity of 6 rev/s, a flowrate of 0.28 m^3/s, and a head of 6 m.

FIND What type of turbine should be selected? Explain.

SOLUTION

The most efficient type of turbine to use can be obtained by calculating the specific speed, N'_{sd}, and using the information of Fig. 12.32. To use the dimensional form of the specific speed indicated in Fig. 12.32, we must convert the given data into the appropriate units. For the rotor speed we get

$$\omega = 6 \text{ rev/s} \times 2 \text{ pi radian per revolution} = 37.7 \text{ rad/s}$$

To estimate the shaft power, we assume all of the available head is converted into power and multiply this amount by an assumed efficiency (94%).

$$\dot{W}_{\text{shaft}} = \gamma Q z \eta = (9800 \text{ N/m}^3)(0.28 \text{ m}^3/\text{s})\left[\frac{6 \text{ m}(0.94)}{1 \text{ N}\cdot\text{m/s}\cdot\text{W}}\right]$$

$$\dot{W}_{\text{shaft}} = 15.5 \text{ kW}$$

Thus for this turbine,

$$N'_{sd} = \frac{\omega\sqrt{\dot{W}_{\text{shaft}}}}{(h_a)^{5/4}} = \frac{(37.7 \text{ rad/s})\sqrt{15.5 \text{ kW}}}{(6 \text{ m})^{5/4}} = 0.9$$

According to the information from Fig. 12.32,

A mixed-flow Francis turbine would probably give the highest efficiency and an assumed efficiency of 0.94 is appropriate. **(Ans)**

COMMENT What would happen if we wished to use a Pelton wheel for this application? Note that with only a 6.0 m head, the maximum jet velocity, V_1, obtainable (neglecting viscous effects) would be

$$V_1 = \sqrt{2\,gz} = \sqrt{2 \times 9.8 \text{ m/s}^2 \times 6.0 \text{ m}} = 10.8 \text{ m/s}$$

As shown by Eq. 12.52, for maximum efficiency of a Pelton wheel the jet velocity is ideally two times the blade velocity. Thus, $V_1 = 2\omega R$, or the wheel diameter, $D = 2R$, is

$$D = \frac{V_1}{\omega} = \frac{10.8 \text{ m/s}}{(6 \text{ rev/s} \times 2\pi \text{ rad/rev})} = 0.29 \text{ m}$$

To obtain a flowrate of $Q = 0.28 \text{ m}^3/\text{s}$ at a velocity of $V_1 = 10.8$ m/s, the jet diameter, d_1, must be given by

$$Q = \frac{\pi}{4} d_1^2 V_1$$

or

$$d_1 = \left[\frac{4Q}{\pi V_1}\right]^{1/2} = \left[\frac{4(0.28 \text{ m}^3/\text{s})}{\pi(10.8 \text{ m/s})}\right]^{1/2} = 0.18 \text{ m}$$

A Pelton wheel with a diameter of $D = 0.29$ m supplied with water through a nozzle of diameter $d_1 = 0.18$ m is not a practical design. Typically $d_1 << D$ (see Fig. 12.22). By using multiple jets it would be possible to reduce the jet diameter. However, even with 8 jets, the jet diameter would be 0.06 m, which is still too large (relative to the wheel diameter) to be practical. Hence, the above calculations reinforce the results presented in Fig. 12.32—a Pelton wheel would not be practical for this application. If the flowrate were considerably smaller, the specific speed could be reduced to the range where a Pelton wheel would be the type to use (rather than a mixed-flow reaction turbine).

12.9 Compressible Flow Turbomachines

Compressible flow turbomachines are in many ways similar to the incompressible flow pumps and turbines described in previous portions of this chapter. The main difference is that the density of the fluid (a gas or vapor) changes significantly from the inlet to the outlet of the compressible flow machines. This added feature has interesting consequences, benefits, and complications.

Compressors are pumps that add energy to the fluid, causing a significant pressure rise and a corresponding significant increase in density. Compressible flow turbines, on the other hand, remove energy from the fluid, causing a lower pressure and a smaller density at the outlet than at the inlet. The information provided earlier about basic energy considerations (Section 12.2) and basic angular momentum considerations (Section 12.3) is directly applicable to these turbomachines in the ways demonstrated earlier.

As discussed in Chapter 11, compressible flow study requires an understanding of the principles of thermodynamics. Similarly, an in-depth analysis of compressible flow turbomachines requires use of various thermodynamic concepts. In this section we provide only a brief discussion of some of the general properties of compressors and compressible flow turbines. The interested reader is encouraged to read some of the excellent references available for further information (e.g., Refs. 1–3, 18–20).

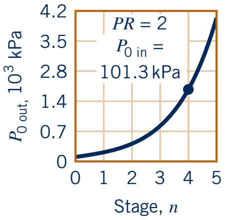
12.9.1 Compressors

Turbocompressors operate with the continuous compression of gas flowing through the device. Since there is a significant pressure and density increase, there is also a considerable temperature increase.

Radial-flow (or centrifugal) compressors are essentially centrifugal pumps (see Section 12.4) that use a gas (rather than a liquid) as the working fluid. They are typically high pressure rise, low flowrate, and axially compact turbomachines. A photograph of the rotor of a centrifugal compressor rotor is shown in Fig. 12.33.

The amount of compression is typically given in terms of the *total pressure ratio*, $PR = p_{02}/p_{01}$, where the pressures are absolute. Thus, a radial-flow compressor with $PR = 3.0$ can compress standard atmospheric air from 101.3 kPa to 3.0×101.3 kPa $= 304$ kPa.

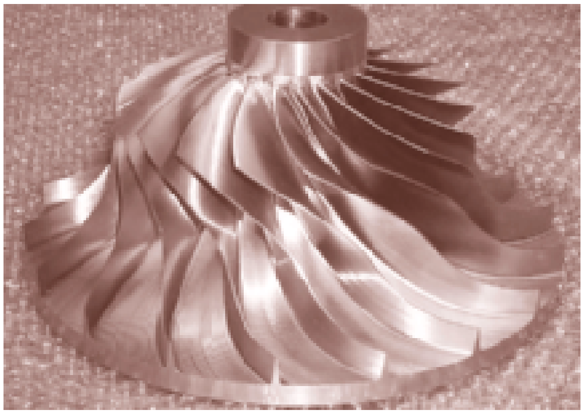
Multistaging is common in high-pressure ratio compressors.



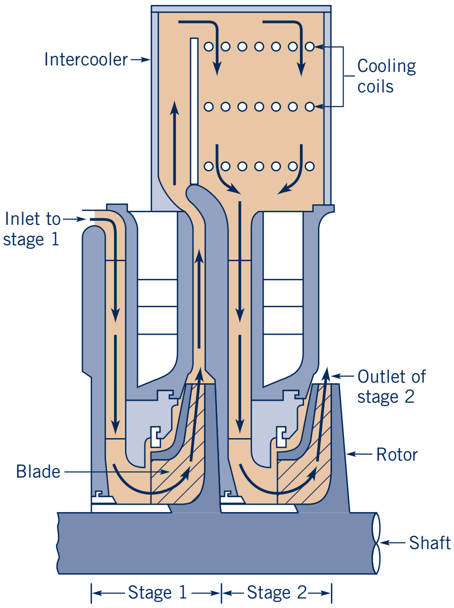
Higher pressure ratios can be obtained by using *multiple-stage* devices in which flow from the outlet of the preceding stage proceeds to the inlet of the following stage. If each stage has the same pressure ratio, PR, the overall pressure ratio after n stages is PR^n. Thus, as shown by the figure in the margin, a four-stage compressor with individual stage $PR = 2.0$ can compress standard air from $p_{0\,in} = 101.3$ kPa to $p_{0\,out} = 2^4 \times 101.3$ kPa $= 1620$ kPa. Adiabatic (i.e., no heat transfer) compression of a gas causes an increase in temperature and requires more work than isothermal (constant temperature) compression of a gas. An interstage cooler (i.e., an intercooler heat exchanger) as shown in Fig. 12.34 can be used to reduce the compressed gas temperature and thus the work required.

Relative to centrifugal water pumps, radial compressors of comparable size rotate at much higher speeds. It is not uncommon for the rotor blade exit speed and the speed of the absolute flow leaving the impeller to be greater than the speed of sound. That such large speeds are necessary for compressors can be seen by noting that the large pressure rise designed for is related to the differences of several squared speeds (see Eq. 12.14).

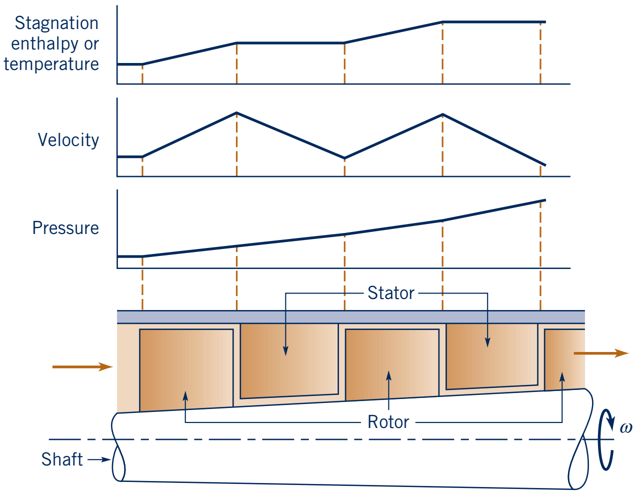
The axial-flow compressor is the other widely used configuration. This type of turbomachine has a lower pressure rise per stage and a higher flowrate, and is more radially compact than a centrifugal compressor. As shown in Fig. 12.35, axial-flow compressors usually consist of several stages, with each stage containing a rotor/stator row pair. For an 11-stage compressor, a compression ratio of $PR = 1.2$ per stage gives an overall pressure ratio of $p_{02}/p_{01} = 1.2^{11} = 7.4$. As the



■ **Figure 12.33** **Centrifugal compressor rotor. (Photograph courtesy of Concepts NREC.)**



■ **Figure 12.34** **Two-stage centrifugal compressor with an intercooler.**



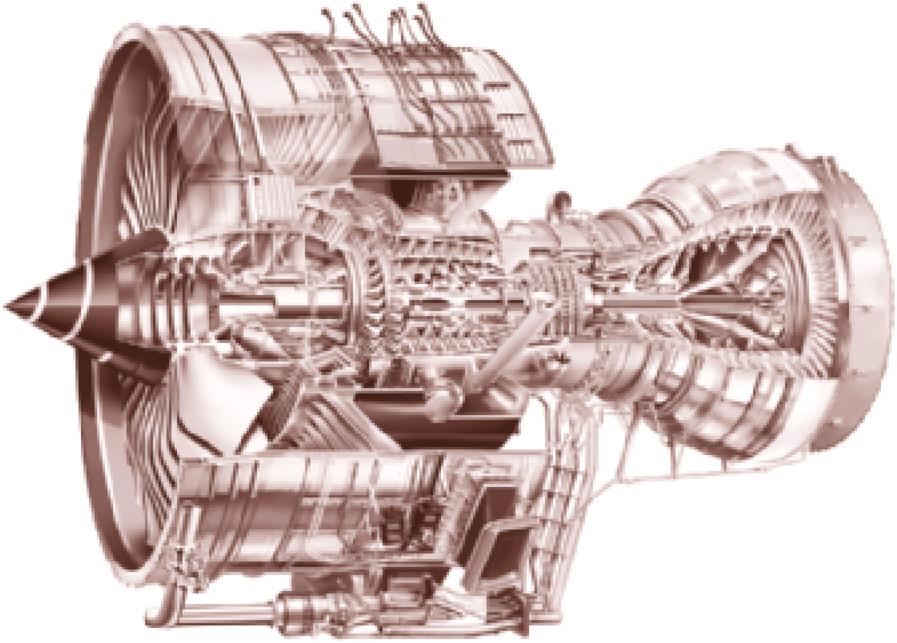
■ **Figure 12.35** **Enthalpy, velocity, and pressure distribution in an axial-flow compressor.**

Axial-flow compressor multistaging requires less space than centrifugal compressors.

gas is compressed and its density increases, a smaller annulus cross-sectional area is required and the flow channel size decreases from the inlet to the outlet of the compressor. The typical jet aircraft engine uses an axial-flow compressor as one of its main components (see Fig. 12.36 and Ref. 21).

An axial-flow compressor can include a set of *inlet guide vanes* upstream of the first rotor row. These guide vanes optimize the size of the relative velocity into the first rotor row by directing the flow away from the axial direction. *Rotor blades* push on the gas in the direction of blade motion and to the rear, adding energy (like in an axial pump) and moving the gas through the compressor. The *stator blade* rows act as diffusers, turning the fluid back toward the axial direction and increasing the static pressure. The stator blades cannot add energy to the fluid because they are stationary. Typical pressure, velocity, and enthalpy distributions along the axial direction are shown in Fig. 12.35. [If you

V12.6 Flow in a compressor stage



■ **Figure 12.36** **Rolls-Royce Trent 900 three-shaft propulsion system. (Courtesy of Rolls-Royce plc.)**

are not familiar with the thermodynamic concept of enthalpy (see Section 11.1), you may replace "enthalpy" by temperature as an approximation.] The reaction of the compressor stage is equal to the ratio of the rise in static enthalpy or temperature achieved across the rotor to the enthalpy or temperature rise across the stage. Most modern compressors involve 50% or higher reaction.

Compressor blades can stall, and unstable flow conditions can subsequently occur.

The blades in an axial-flow compressor are airfoils carefully designed to produce appropriate lift and drag forces on the flowing gas. As occurs with airplane wings, compressor blades can stall (see Section 9.4). When the flowrate is decreased from the design amount, the velocity triangle at the entrance of the rotor row indicates that the relative flow meets the blade leading edge at larger angles of incidence than the design value. When the angle of incidence becomes too large, blade stall can occur and the result is *compressor surge* or *stall*—unstable flow conditions that can cause excessive vibration, noise, poor performance, and possible damage to the machine. The lower flowrate bound of compressor operation is related to the beginning of these instabilities (see Fig. 12.37).

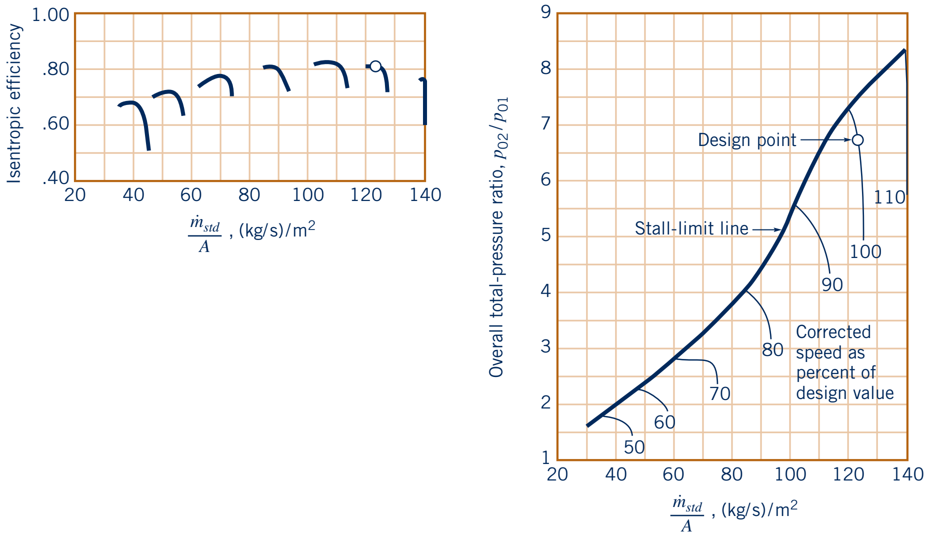
Other important compressible flow phenomena such as variations of the Mach cone (see Section 11.3), shock waves (see Section 11.5.3), and choked flow (see Section 11.4.2) occur commonly in compressible flow turbomachines. They must be carefully designed for. These phenomena are very sensitive to even very small changes or variations of geometry. Shock strength is kept low to minimize shock loss, and choked flows limit the upper flowrate boundary of machine operation (see Fig. 12.37).

The experimental performance data for compressors are systematically summarized with parameters prompted by dimensional analysis. As mentioned earlier, total pressure ratio, p_{02}/p_{01}, is used instead of the head-rise coefficient associated with pumps, blowers, and fans.

Either isentropic or polytropic efficiencies are used to characterize compressor performance. A detailed explanation of these efficiencies is beyond the scope of this text. Those interested in learning more about these parameters should study any of several available books on turbomachines (for example, Refs. 2 and 3). Basically, each of these compressor efficiencies involves a ratio of ideal work to actual work required to accomplish the compression. The isentropic efficiency involves a ratio of the ideal work required with an adiabatic and frictionless (no loss) compression process to the actual work required to achieve the same total pressure rise. The polytropic efficiency involves a ratio of the ideal work required to achieve the actual end state of the compression with a polytropic and frictionless process between the actual beginning and end stagnation states across the compressor and the actual work involved between these same states.

The flow parameter commonly used for compressors is based on the following dimensionless grouping from dimensional analysis

$$\frac{R\dot{m}\sqrt{kRT_{01}}}{D^2\,p_{01}}$$



■ **Figure 12.37** **Performance characteristics of an axial-flow compressor (Ref. 19).**

where R is the gas constant, $\dot{m}$ the mass flowrate, k the specific heat ratio, T_{01} the stagnation temperature at the compressor inlet, D a characteristic length, and p_{01} the stagnation pressure at the compressor inlet.

To account for variations in test conditions, the following strategy is employed. We set

$$\left(\frac{R\dot{m}\sqrt{kRT_{01}}}{D^2\,p_{01}}\right)_{\text{test}} = \left(\frac{R\dot{m}\sqrt{kRT_{01}}}{D^2\,p_{01}}\right)_{\text{std}}$$

where the subscript "test" refers to a specific test condition and "std" refers to the standard atmosphere ($p_0 = 101.3$ kPa, $T_0 = 288$ K) condition. When we consider a given compressor operating on a given working fluid (so that R, k, and D are constant), the above equation reduces to

$$\dot{m}_{\text{std}} = \frac{\dot{m}_{\text{test}}\sqrt{T_{01\ \text{test}}/T_{0\ \text{std}}}}{p_{01\ \text{test}}/p_{0\ \text{std}}} \qquad \textbf{(12.54)}$$

In essence, $\dot{m}_{\text{std}}$ is the compressor-test mass flowrate "corrected" to the standard atmosphere inlet condition. *The corrected compressor mass flowrate*, $\dot{m}_{\text{std}}$, is used instead of flow coefficient. Often, $\dot{m}_{\text{std}}$ is divided by A, the frontal area of the compressor flow path.

While for pumps, blowers, and fans, rotor speed was accounted for in the flow coefficient, it is not in the corrected mass flowrate derived above. Thus, for compressors, rotor speed needs to be accounted for with an additional group. This dimensionless group is

$$\frac{ND}{\sqrt{kRT_{01}}}$$

For the same compressor operating on the same gas, we eliminate D, k, and R and, as with corrected mass flowrate, obtain a corrected speed, N_{std}, where

$$N_{\text{std}} = \frac{N}{\sqrt{T_{01}/T_{\text{std}}}} \qquad \textbf{(12.55)}$$

Often, the percentage of the corrected speed design value is used.

An example of how compressor performance data are typically summarized is shown in Fig. 12.37.

12.9.2 Compressible Flow Turbines

A gas turbine engine generally consists of a compressor, a combustor, and a turbine.

Turbines that use a gas or vapor as the working fluid are in many respects similar to hydraulic turbines (see Section 12.8). Compressible flow turbines may be impulse or reaction turbines, and mixed-, radial-, or axial-flow turbines. The fact that the gas may expand (compressible flow) in coursing through the turbine can introduce some important phenomena that do not occur in hydraulic turbines. (*Note:* It is tempting to label turbines that use a gas as the working fluid as gas turbines. However, the terminology "gas turbine" is commonly used to denote a *gas turbine engine*, as employed, for example, for aircraft propulsion or stationary power generation. As shown in Fig. 12.36, these engines typically contain a compressor, combustion chamber, and turbine.)

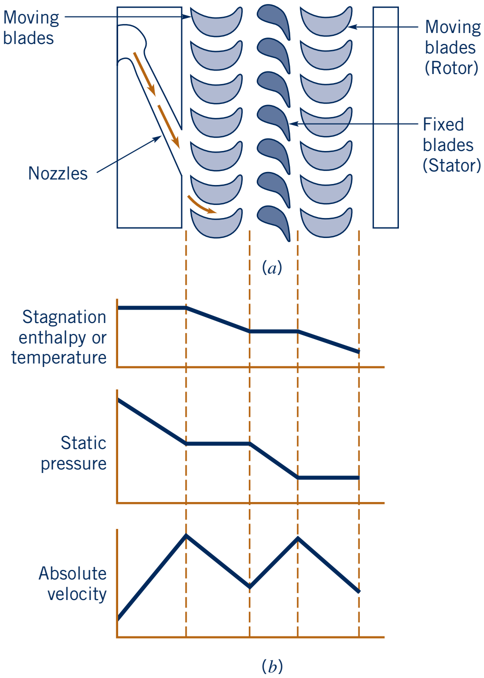
Although for compressible flow turbines the axial-flow type is common, the radial-inflow type is also used for various purposes. As shown in Fig. 12.33, the turbine that drives the typical automobile turbocharger compressor is a radial-inflow type. The main advantages of the radial-inflow turbine are: (1) It is robust and durable, (2) it is axially compact, and (3) it can be relatively inexpensive. A radial-flow turbine usually has a lower efficiency than an axial-flow turbine, but lower initial costs may be the compelling incentive in choosing a radial-flow turbine over an axial-flow one.

Axial-flow turbines are widely used compressible flow turbines. Steam engines used in electrical generating plants and marine propulsion and the turbines used in gas turbine engines are usually of the axial-flow type. Often they are multistage turbomachines, although single-stage compressible turbines are also produced. They may be either an impulse type or a reaction type.

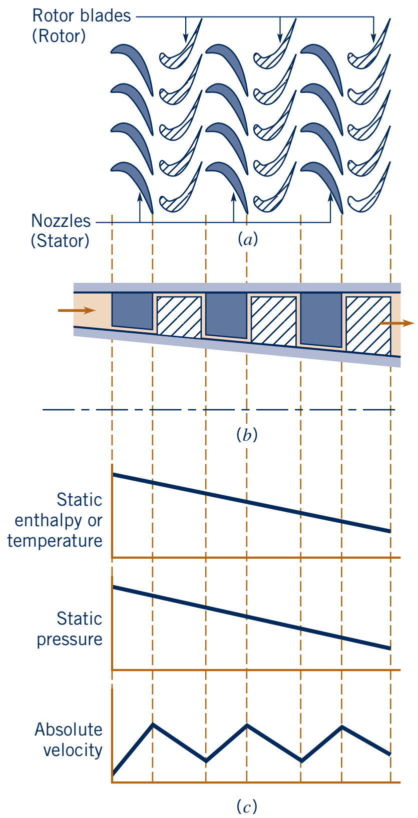
With compressible flow turbines, the ratio of static enthalpy or temperature drop across the rotor to this drop across the stage, rather than the ratio of static pressure differences, is used to determine reaction. Strict impulse (zero pressure drop) turbines have slightly negative reaction; the static enthalpy or temperature actually increases across the rotor. Zero-reaction turbines involve no change of static enthalpy or temperature across the rotor but do involve a slight pressure drop.

A two-stage, axial-flow impulse turbine is shown in Fig. 12.38*a*. The gas accelerates through the supply nozzles, has some of its energy removed by the first-stage rotor blades, accelerates again through the second-stage nozzle row, and has additional energy removed by the second-stage rotor blades. As shown in Fig. 12.38*b*, the static pressure remains constant across the rotor rows. Across the second-stage nozzle row, the static pressure decreases, absolute velocity increases, and the stagnation enthalpy (temperature) is constant. Flow across the second rotor is similar to flow across the first rotor. Since the working fluid is a gas, the significant decrease in static pressure across the turbine results in a significant decrease in density—the flow is compressible. Hence, more detailed analysis of this flow must incorporate various compressible flow concepts developed in Chapter 11. Interesting phenomena such as shock waves and choking due to sonic conditions at the "throat" of the flow passage between blades can occur because of compressibility effects. The interested reader is encouraged to consult the various references available (e.g., Refs. 2, 3, 20) for fascinating applications of compressible flow principles in turbines.

The rotor and nozzle blades in a three-stage, axial-flow reaction turbine are shown in Fig. 12.39*a*. The axial variations of pressure and velocity are shown in Fig. 12.39*c*. Both the stationary and rotor blade (passages) act as flow-accelerating nozzles. That is, the static pressure and enthalpy (temperature) decrease in the direction of flow for both the fixed and the rotating blade rows. This distinguishes the reaction turbine from the impulse turbine (see Fig. 12.38*b*). Energy is removed from the fluid by the rotors only (the stagnation enthalpy or temperature is constant across the adiabatic flow stators).



■ **Figure 12.38 Enthalpy, velocity, and pressure distribution in a two-stage impulse turbine.**



■ **Figure 12.39 Enthalpy, pressure, and velocity distribution in a three-stage reaction turbine.**

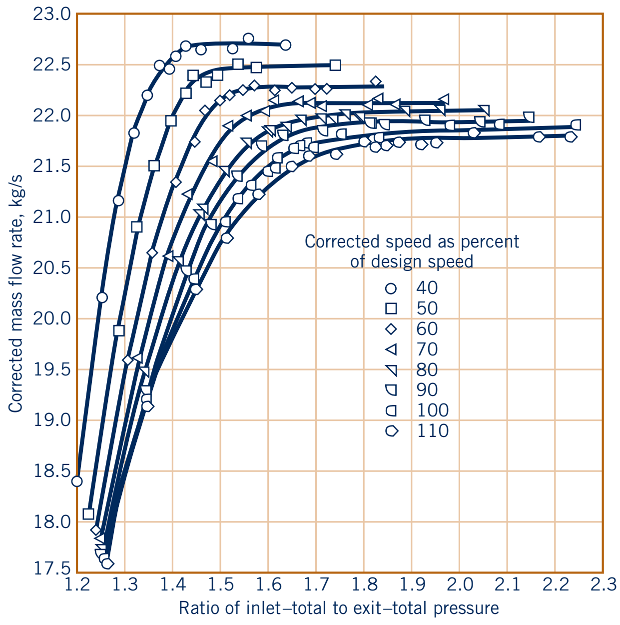


Figure 12.40 **Typical compressible flow turbine performance "map." (Ref. 20)**

Because of the reduction of static pressure in the downstream direction, the gas expands, and the flow passage area must increase from the inlet to the outlet of this turbine. This is seen in Fig. 12.39*b*.

Turbine performance maps are used to display complex turbine characteristics.

Performance data for compressible flow turbines are summarized with the help of parameters derived from dimensional analysis. Isentropic and polytropic efficiencies (see Refs. 2, 3, and 20) are commonly used, as are inlet-to-outlet total pressure ratios (p_{01}/p_{02}), corrected rotor speed (see Eq. 12.55), and corrected mass flowrate (see Eq. 12.54). Figure 12.40 shows a compressible flow turbine performance "map."

12.10 Chapter Summary and Study Guide

turbomachine
axial-, mixed-, and radial-flow
velocity triangle
angular momentum
shaft torque
Euler turbomachine equation
shaft power
centrifugal pump
pump performance curve
overall efficiency
system equation
head-rise coefficient
power coefficient
flow coefficient
pump scaling laws
specific speed
impulse turbine
reaction turbine
Pelton wheel

Various aspects of turbomachine flow are considered in this chapter. The connection between fluid angular momentum change and shaft torque is key to understanding how turbo-pumps and turbines operate.

The shaft torque associated with change in the axial component of angular momentum of a fluid as it flows through a pump or turbine is described in terms of the inlet and outlet velocity triangle diagrams. Such diagrams indicate the relationship among absolute, relative, and blade velocities.

Performance characteristics for centrifugal pumps are discussed. Standard dimensionless pump parameters, similarity laws, and the concept of specific speed are presented for use in pump analysis. How to use pump performance curves and the system curve for proper pump selection is presented. A brief discussion of axial-flow and mixed-flow pumps is given.

An analysis of impulse turbines is provided, with emphasis on the Pelton wheel turbine. For impulse turbines there is negligible pressure difference across the blade; the torque is a result of the change in direction of the fluid jet striking the blade. Radial-flow and axial-flow reaction turbines are also briefly discussed.

The following checklist provides a study guide for this chapter. When your study of the entire chapter and end-of-chapter exercises has been completed, you should be able to

- write out meanings of the terms listed here in the margin and understand each of the related concepts. These terms are particularly important and are set in ***italic, bold, and color*** type in the text.
- draw appropriate velocity triangles for flows entering and leaving given pump or turbine configurations.

- estimate the actual shaft torque, actual shaft power, and ideal pump head rise for a given centrifugal pump configuration.
- use pump performance curves and the system curve to predict pump performance in a given system.
- predict the performance characteristics for one pump based on the performance of another pump of the same family using the pump scaling laws.
- use specific speed to determine whether a radial-flow, mixed-flow, or axial-flow pump would be most appropriate for a given situation.
- estimate the actual shaft torque and actual shaft power for flow through an impulse turbine configuration.
- estimate the actual shaft torque and actual shaft power for a given reaction turbine.
- use specific speed to determine whether an impulse or a reaction turbine would be most appropriate for a given situation.

Some of the important equations in this chapter are:

Vector addition of velocities	$\mathbf{V} = \mathbf{W} + \mathbf{U}$	**(12.1)**
Shaft torque	$T_{\text{shaft}} = -\dot{m}_1(r_1 V_{\theta 1}) + \dot{m}_2(r_2 V_{\theta 2})$	**(12.2)**
Shaft power	$\dot{W}_{\text{shaft}} = T_{\text{shaft}}\,\omega$	**(12.3)**
Shaft power	$\dot{W}_{\text{shaft}} = -\dot{m}_1(U_1 V_{\theta 1}) + \dot{m}_2(U_2 V_{\theta 2})$	**(12.4)**
Shaft work	$w_{\text{shaft}} = U_2 V_{\theta 2} - U_1 V_{\theta 1}$	**(12.12)**
Pump ideal head rise	$h_i = \dfrac{U_2 V_{\theta 2} - U_1 V_{\theta 1}}{g}$	**(12.13)**
Pump actual head rise	$h_a = \dfrac{p_2 - p_1}{\gamma} + z_2 - z_1 + \dfrac{V_2^2 - V_1^2}{2g}$	**(12.19)**
Pump similarity relationship	$\dfrac{gh_a}{\omega^2 D^2} = \phi_1\left(\dfrac{Q}{\omega D^3}\right)$	**(12.29)**
Pump similarity relationship	$\dfrac{\dot{W}_{\text{shaft}}}{\rho\omega^3 D^5} = \phi_2\left(\dfrac{Q}{\omega D^3}\right)$	**(12.30)**
Pump similarity relationship	$\eta = \phi_3\left(\dfrac{Q}{\omega D^3}\right)$	**(12.31)**
Pump scaling law	$\left(\dfrac{Q}{\omega D^3}\right)_1 = \left(\dfrac{Q}{\omega D^3}\right)_2$	**(12.32)**
Pump scaling law	$\left(\dfrac{gh_a}{\omega^2 D^2}\right)_1 = \left(\dfrac{gh_a}{\omega^2 D^2}\right)_2$	**(12.33)**
Pump scaling law	$\left(\dfrac{\dot{W}_{\text{shaft}}}{\rho\omega^3 D^5}\right)_1 = \left(\dfrac{\dot{W}_{\text{shaft}}}{\rho\omega^3 D^5}\right)_2$	**(12.34)**
Pump scaling law	$\eta_1 = \eta_2$	**(12.35)**
Specific speed (pumps)	$N_{sd} = \dfrac{\omega(\text{rpm})\sqrt{Q(\text{L/min})}}{[h_a(\text{m})]^{3/4}}$	**(12.44)**
Specific speed (turbines)	$N'_{sd} = \dfrac{\omega(\text{rpm})\sqrt{\dot{W}_{\text{shaft}}\,(\text{W})}}{[h_a(\text{m})]^{5/4}}$	**(12.53)**

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Problem available in *WileyPLUS* at instructor's discretion.

GO Tutoring problem available in *WileyPLUS* at instructor's discretion.

Problem is related to a chapter video available in *WileyPLUS*.

* Problem to be solved with aid of programmable calculator or computer.

† Open-ended problem that requires critical thinking. These problems require various assumptions to provide the necessary input data. There are not unique answers to these problems.

Review Problems

Go to Appendix G (*WileyPLUS* or the book's website, www.wiley.com/college/munson) for a set of review problems with answers. Detailed solutions can be found in the *Student Solution Manual and Study Guide for Fundamentals of Fluid Mechanics*, by Munson et al. (© 2013 John Wiley and Sons, Inc.).

Conceptual Questions

12.1C For a turbomachine pump, such as a window fan or a propeller,

a) rotation of the fan or propeller results in movement of fluid.

b) rotation of the fan or propeller results in energy being transferred to the fluid.

c) rotation of the fan or propeller requires work input to the fan or propeller shaft.

d) All of the above.

12.2C A purpose of a water pump is to turn the pump shaft work into an increase in

a) the pressure of the fluid. b) the volume of the fluid.
c) the density of the fluid. d) the enthalpy of the fluid.

12.3C When wind is acting on a windmill, the power produced will depend on

a) the velocity of the wind relative to the blade.
b) the velocity of the blade relative to the ground.
c) the velocity of the ground relative to the blade.
d) the velocity of the wind relative to the ground.

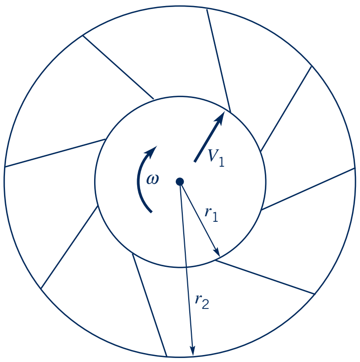
Additional conceptual questions are available in *WileyPLUS* at the instructor's discretion.

Problems

Note: Unless specific values of required fluid properties are given in the problem statement, use the values found in the tables on the inside of the front cover. Answers to the even-numbered problems are listed at the end of the book. The Lab Problems as well as the videos that accompany problems can be accessed in *WileyPLUS* or the book's website, www.wiley.com/college/munson.

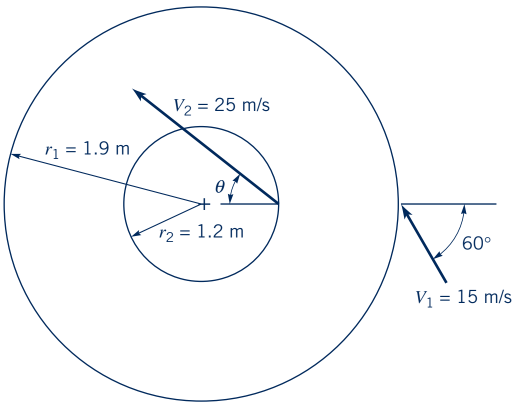
Section 12.1 Introduction and Section 12.2 Basic Energy Considerations

12.1 The rotor shown in Fig. P12.1 rotates clockwise. Assume that the fluid enters in the radial direction and the relative velocity is tangent to the blades and remains constant across the entire rotor. Is the device a pump or a turbine? Explain.



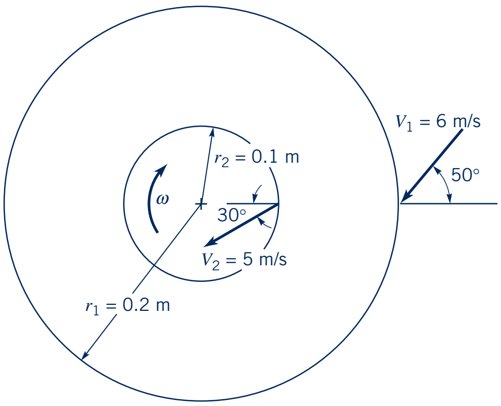
■ **Figure P12.1**

12.2 Air (assumed incompressible) flows across the rotor shown in Fig. P12.2 such that the magnitude of the absolute velocity increases from 15 m/s to 25 m/s. Measurements indicate that the absolute velocity at the inlet is in the direction shown. Determine the direction of the absolute velocity at the outlet if the fluid puts no torque on the rotor. Is the rotation CW or CCW? Is this device a pump or a turbine?



■ **Figure P12.2**

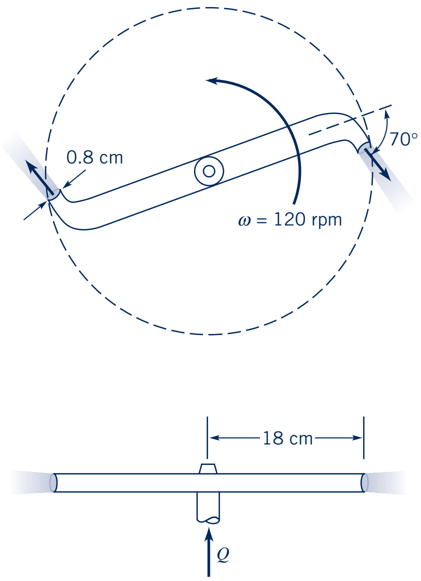
12.3 The measured shaft torque on the turbomachine shown in Fig. P12.3 is −60 N·m when the absolute velocities are as indicated. Determine the mass flowrate. What is the angular velocity if the magnitude of the shaft power is 1800 N·m/s? Is this machine a pump or a turbine? Explain.



■ **Figure P12.3**

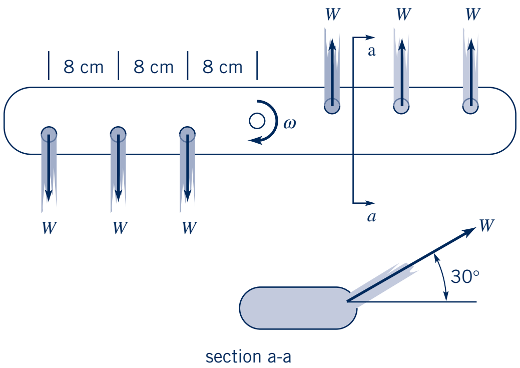
Section 12.3 Basic Angular Momentum Considerations

12.4 Water flows through a rotating sprinkler arm as shown in Fig. P12.4 and **Video V12.2.** Estimate the minimum water pressure necessary for an angular velocity of 150 rpm. Is this a turbine or a pump?



■ **Figure P12.4**

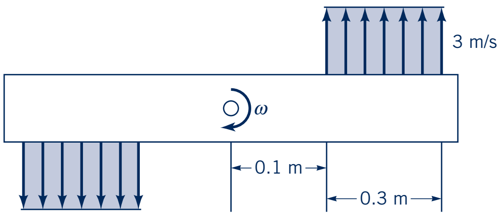
12.5 Water is supplied to a dishwasher through the manifold shown in Fig. P12.5. Determine the rotational speed of the manifold if bearing friction and air resistance are neglected. The total flowrate of 9 L/pm is divided evenly among the six outlets, each of which produces a 4/5 cm diameter stream.



■ **Figure P12.5**

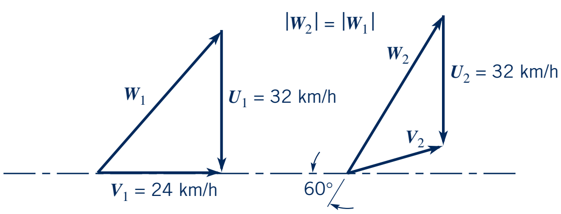
12.6 Water flows axially up the shaft and out through the two sprinkler arms as sketched in Fig. P12.4 and as shown in **Video V12.2.** With the help of the moment-of-momentum equation explain why only at a threshold amount of water flow, the sprinkler arms begin to rotate. What happens when the flowrate increases above this threshold amount? If the exit nozzle could be varied, what would happen for a set flowrate above the threshold amount, when the angle is increased to 90°? Decreased to 0°?

12.7 GO Uniform horizontal sheets of water of 3 mm thickness issue from the slits on the rotating manifold shown in Fig. P12.7. The velocity relative to the arm is a constant 3 m/s along each slit. Determine the torque needed to hold the manifold stationary. What would the angular velocity of the manifold be if the resisting torque is negligible?



■ **Figure P12.7**

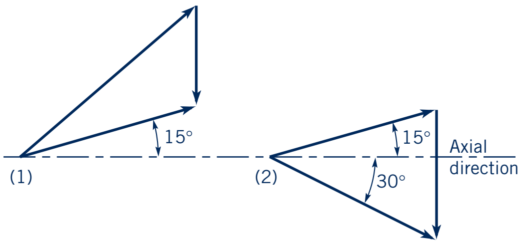
12.8 At a given radial location, a 24 km/h wind against a windmill (**see Video V12.1**) results in the upstream (1) and downstream (2) velocity triangles shown in Fig. P12.8. Sketch an appropriate blade section at that radial location and determine the energy transferred per unit mass of fluid.



■ **Figure P12.8**

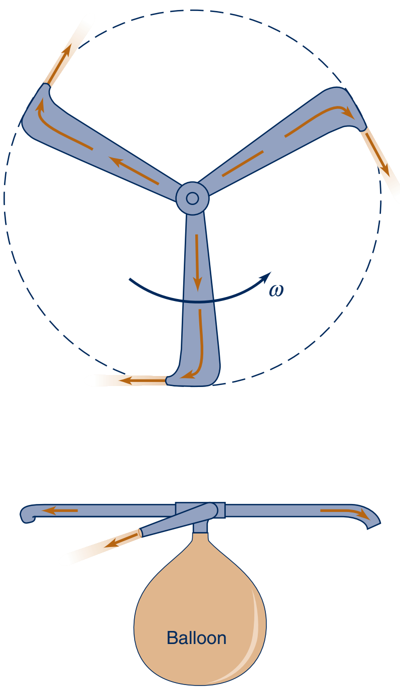
12.9 Sketch how you would arrange four 8 cm wide by 30.5 cm long thin but rigid strips of sheet metal on a hub to create a windmill like the one shown in **Video V12.1.** Discuss, with the help of velocity triangles, how you would arrange each blade on the hub and how you would orient your windmill in the wind.

12.10 Sketched in Fig. P12.10 are the upstream [section (1)] and downstream [section (2)] velocity triangles at the arithmetic mean radius for flow through an axial-flow turbomachine rotor. The axial component of velocity is 15 m/s at sections (1) and (2). **(a)** Label each velocity vector appropriately. Use **V** for absolute velocity, **W** for relative velocity, and **U** for blade velocity. **(b)** Are you dealing with a turbine or a fan? **(c)** Calculate the work per unit mass involved. **(d)** Sketch a reasonable blade section. Do you think that the actual blade exit angle will need to be less or greater than 15°? Why?



■ **Figure P12.10**

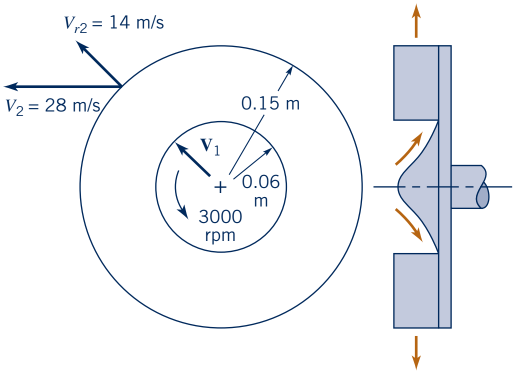
12.11 Shown in Fig. P12.11 is a toy "helicopter" powered by air escaping from a balloon. The air from the balloon flows radially through each of the three propeller blades and out small nozzles at the tips of the blades. The nozzles (along with the rotating propeller blades) are tilted at a small angle as indicated. Sketch the velocity triangle (i.e., blade, absolute, and relative velocities) for the flow from the nozzles. Explain why this toy tends to move upward. Is this a turbine? Pump?



■ **Figure P12.11**

Section 12.4 The Centrifugal Pump and Section 12.4.1 Theoretical Considerations

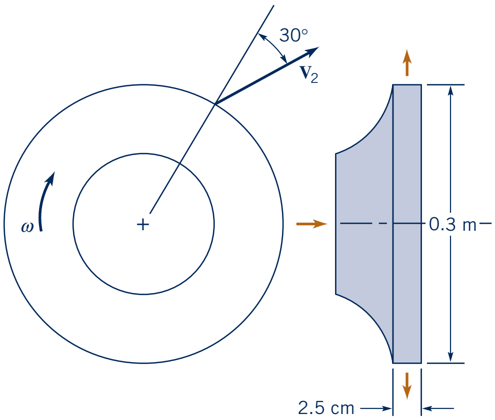
12.12 The radial component of velocity of water leaving the centrifugal pump sketched in Fig. P12.12 is 14 m/s. The magnitude of the absolute velocity at the pump exit is 28 m/s. The fluid enters the pump rotor radially. Calculate the shaft work required per unit mass flowing through the pump.



■ **Figure P12.12**

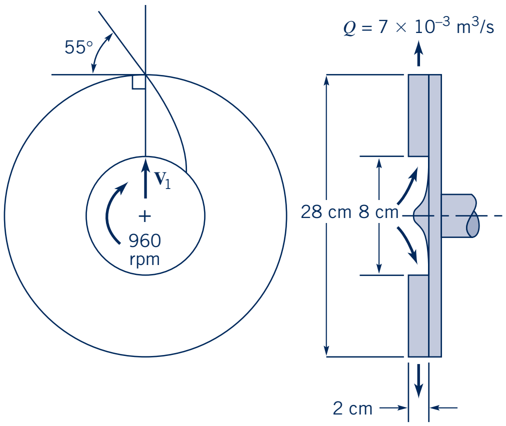
12.13 A centrifugal water pump having an impeller diameter of 0.5 m operates at 900 rpm. The water enters the pump parallel to the pump shaft. If the exit blade angle, β_2 (see Fig. 12.8), is 25°, determine the shaft power required to turn the impeller when the flow through the pump is 0.16 m³/s. The uniform blade height is 50 mm.

12.14 GO A centrifugal pump impeller is rotating at 1200 rpm in the direction shown in Fig. P12.14. The flow enters parallel to the axis of rotation and leaves at an angle of 30° to the radial direction. The absolute exit velocity, V_2, is 28 m/s. **(a)** Draw the velocity triangle for the impeller exit flow. **(b)** Estimate the torque necessary to turn the impeller if the fluid is water. What will the impeller rotation speed become if the shaft breaks?



■ **Figure P12.14**

12.15 A centrifugal radial water pump has the dimensions shown in Fig. P12.15. The volume rate of flow is 7.0×10^{-3} m³/s, and the absolute inlet velocity is directed radially outward. The angular velocity of the impeller is 960 rpm. The exit velocity as seen from a coordinate system attached to the impeller can be assumed to be tangent to the vane at its trailing edge. Calculate the power required to drive the pump.



■ **Figure P12.15**

Section 12.4.2 Pump Performance Characteristics

12.16 Water is pumped with a centrifugal pump, and measurements made on the pump indicate that for a flowrate of 1 kL/min the required input power is 4476 W. For a pump efficiency of 62%, what is the actual head rise of the water being pumped?

12.17 The performance characteristics of a certain centrifugal pump are determined from an experimental setup similar to that shown in Fig. 12.10. When the flowrate of a liquid ($SG = 0.9$) through the pump is 450 L/min, the pressure gage at (1) indicates a vacuum of 95 mm of mercury and the pressure gage at (2) indicates a pressure of 80 kPa. The diameter of the pipe at the inlet is 110 mm and at the exit it is 55 mm. If $z_2 - z_1 = 0.5$ m, what is the actual head rise across the pump? Explain how you would estimate the pump motor power requirement.

12.18 The performance characteristics of a certain centrifugal pump having a 0.2 cm diameter impeller and operating at 1750 rpm are determined using an experimental setup similar to that shown in Fig. 12.10. The following data were obtained during a series of tests in which $z_2 - z_1 = 0$, $V_2 = V_1$, and the fluid was water.

Q (L/min)	75	150	225	300	375	450	525
$p_2 - p_1$ (kPa)	277	276	263	250	231	208	178
Power input (kW)	1.2	1.7	2.0	2.2	2.4	2.6	3.0

Based on these data, show or plot how the actual head rise, h_a, and the pump efficiency, η, vary with the flowrate. What is the design flowrate for this pump?

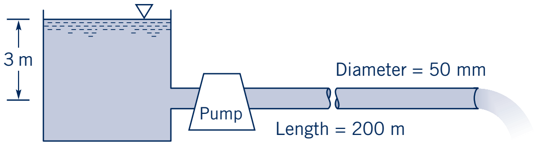
Section 12.4.3 Net Positive Suction Head (NPSH)

12.19 In Example 12.3, how will the maximum height, z_1, that the pump can be located above the water surface change if the water temperature is decreased to 4 °C?

12.20 In Example 12.3, how will the maximum height, z_1, that the pump can be located above the water surface change if **(a)** the water temperature is increased to 48 °C, or **(b)** the fluid is changed from water to gasoline at 15 °C?

12.21 A centrifugal pump with a 18 cm diameter impeller has the performance characteristics shown in Fig. 12.12. The pump is used to pump water at 38 °C, and the pump inlet is located 3.6 m above the open water surface. When the flowrate is 0.8 kL/min, the head loss between the water surface and the pump inlet is 2 m of water. Would you expect cavitation in the pump to be a problem? Assume standard atmospheric pressure. Explain how you arrived at your answer.

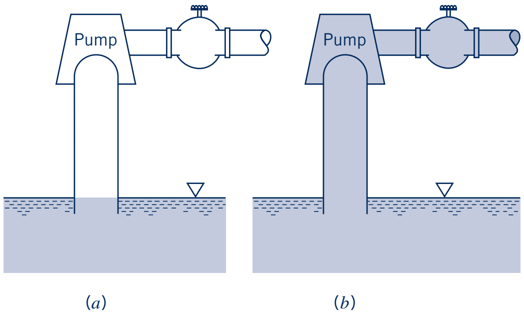
12.22 Water at 40 °C is pumped from an open tank through 200 m of 50 mm diameter smooth horizontal pipe as shown in Fig. P12.22 and discharges into the atmosphere with a velocity of 3 m/s. Minor losses are negligible. **(a)** If the efficiency of the pump is 70%, how much power is being supplied to the pump? **(b)** What is the $NPSH_A$ at the pump inlet? Neglect losses in the short section of pipe connecting the pump to the tank. Assume standard atmospheric pressure.



■ **Figure P12.22**

12.23 A small model of a pump is tested in the laboratory and found to have a specific speed, N_{sd}, equal to 1000 when operating at peak efficiency. Predict the discharge of a larger, geometrically similar pump operating at peak efficiency at a speed of 1800 rpm across an actual head rise of 60 m.

12.24 The centrifugal pump shown in Fig. P12.24 is not self-priming. That is, if the water is drained from the pump and pipe as shown in Fig. P12.24(*a*), the pump will not draw the water into the pump and start pumping when the pump is turned on. However, if the pump is primed [i.e., filled with water as in Fig. P12.24(*b*)], the pump does start pumping water when turned on. Explain this behavior.



■ **Figure P12.24**

Section 12.4.4 System Characteristics and Pump Selection

12.25 Contrast the advantages and disadvantages of using pumps in parallel and in series.

12.26 Owing to fouling of the pipe wall, the friction factor for the pipe of Example 12.4 increases from 0.02 to 0.03. Determine the new flowrate, assuming all other conditions remain the same. What is the pump efficiency at this new flowrate? Explain how a line valve could be used to vary the flowrate through the pipe of Example 12.4. Would it be better to place the valve upstream or downstream of the pump? Why?

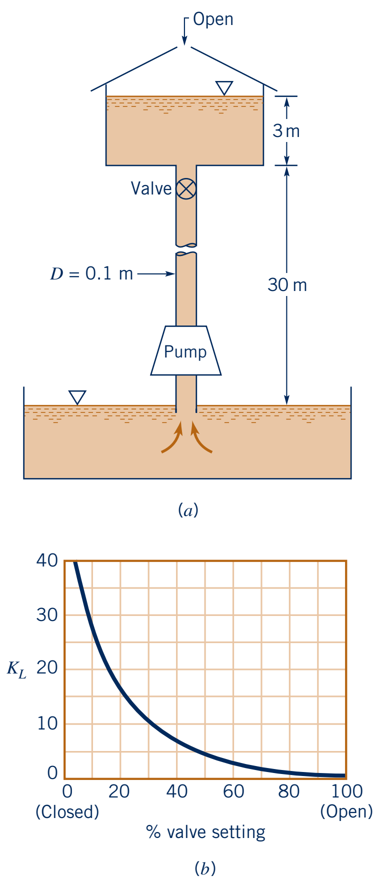
12.27 A centrifugal pump having a head-capacity relationship given by the equation $h_a = 54 - 1.2 \times 10^{-5} Q^2$, with h_a in meters when Q is in L/min, is to be used with a system similar to that shown in Fig. 12.14. For $z_2 - z_1 = 15$ m, what is the expected flowrate if the total length of constant diameter pipe is 182 m and the fluid is water? Assume the pipe diameter to be 10 cm and the friction factor to be equal to 0.02. Neglect all minor losses.

12.28 A centrifugal pump having a 15 cm diameter impeller and the characteristics shown in Fig. 12.12 is to be used to pump gasoline through 1219 m of commercial steel 8 cm diameter pipe. The pipe connects two reservoirs having open surfaces at the same elevation. Determine the flowrate. Do you think this pump is a good choice? Explain.

12.29 Determine the new flowrate for the system described in Problem 12.28 if the pipe diameter is increased from 8 cm to 10 cm. Is this pump still a good choice? Explain.

12.30 A centrifugal pump having the characteristics shown in Example 12.4 is used to pump water between two large open tanks through 30 m of 20 cm diameter pipe. The pipeline contains four regular flanged 90° elbows, a check valve, and a fully open globe valve. Other minor losses are negligible. Assume the friction factor $f = 0.02$ for the 30 m section of pipe. If the static head (difference in height of fluid surfaces in the two tanks) is 9 m, what is the expected flowrate? Do you think this pump is a good choice? Explain.

12.31 In a chemical processing plant a liquid is pumped from an open tank, through a 0.1 m diameter vertical pipe, and into another open tank as shown in Fig. P12.31(*a*). A valve is located in the pipe, and the minor loss coefficient for the valve as a function of the valve setting is shown in Fig. P12.31(*b*). The pump head-capacity relationship is given by the equation $h_a = 52.0 - 1.01 \times 10^3\ Q^2$ with h_a in meters when Q is in m^3/s. Assume the friction factor $f = 0.02$ for the pipe, and all minor losses, except for the valve, are negligible. The fluid levels in the two tanks can be assumed to remain constant. **(a)** Determine the flowrate with the valve wide open. **(b)** Determine the required valve setting (percent open) to reduce the flowrate by 50%.



■ **Figure P12.31**

†**12.32** Water is pumped between the two tanks described in Example 12.4 once a day, 365 days a year, with each pumping period lasting two hours. The water levels in the two tanks remain essentially constant. Estimate the annual cost of the electrical power needed to operate the pump if it were located in your city. You will have to make a reasonable estimate for the efficiency of the motor used to drive the pump. Due to aging, it can be expected that the overall resistance of the system will increase with time. If the operating point shown in Fig. E12.4*c* changes to a point where the flowrate has been reduced to 3780 L/min, what will be the new annual cost of operating the pump? Assume that the cost of electrical power remains the same.

Section 12.5 Dimensionless Parameters and Similarity Laws

12.33 What is the rationale for operating two geometrically similar pumps differing in feature size at the same flow coefficient?

12.34 A centrifugal pump having an impeller diameter of 1 m is to be constructed so that it will supply a head rise of 200 m at a flowrate of 4.1 m^3/s of water when operating at a speed of 1200 rpm. To study the characteristics of this pump, a 1/5 scale, geometrically similar model operated at the same speed is to be tested in the laboratory. Determine the required model discharge and head rise. Assume that both model and prototype operate with the same efficiency (and therefore the same flow coefficient).

12.35 A centrifugal pump with a 30 cm diameter impeller requires a power input of 44,760 W when the flowrate is 12 kL/min against a 18 m head. The impeller is changed to one with a 25 cm diameter. Determine the expected flowrate, head, and input power if the pump speed remains the same.

12.36 Do the head–flowrate data shown in Fig. 12.12 appear to follow the similarity laws as expressed by Eqs. 12.39 and 12.40? Explain.

12.37 A centrifugal pump has the performance characteristics of the pump with the 15 cm diameter impeller described in Fig. 12.12. Note that the pump in this figure is operating at 3500 rpm. What is the expected head gained if the speed of this pump is reduced to 2800 rpm while operating at peak efficiency?

12.38 A centrifugal pump provides a flowrate of 2 kL/min when operating at 1750 rpm against a 60 m head. Determine the pump's flowrate and developed head if the pump speed is increased to 3500 rpm.

12.39 Use the data given in Problem 12.18 and plot the dimensionless coefficients C_H, $C_{\mathcal{P}}$, η versus C_Q for this pump. Calculate a meaningful value of specific speed, discuss its usefulness, and compare the result with data of Fig. 12.18.

12.40 In a certain application, a pump is required to deliver 19 kL/min against a 90 m head when operating at 1200 rpm. What type of pump would you recommend?

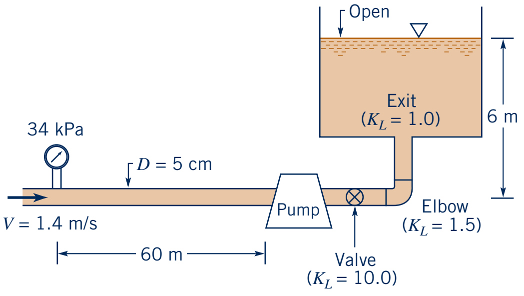
Section 12.6 Axial-Flow and Mixed-Flow Pumps

12.41 Explain how a marine propeller and an axial-flow pump are similar in the main effect they produce.

12.42 A certain axial-flow pump has a specific speed of $N_s = 5.0$. If the pump is expected to deliver 11,340 L/min when operating against a 4.5 m head, at what speed (rpm) should the pump be run?

12.43 A certain pump is known to have a capacity of 3 m^3/s when operating at a speed of 60 rad/s against a head of 20 m. Based on the information in Fig. 12.18, would you recommend a radial-flow, mixed-flow, or axial-flow pump?

12.44 Fuel oil (sp. wt = 7528 N/m^3, viscosity = 95×10^{-5} N · s/m^2) is pumped through the piping system of Fig. P12.44 with a velocity of 1.4 m/s. The pressure 60 m upstream from the pump is 34 kPa. Pipe losses downstream from the pump are negligible, but minor losses are not (minor loss coefficients are given on the figure). **(a)** For a pipe diameter of 5 cm with a relative roughness $\varepsilon/D = 0.001$, determine the head that must be added by the pump. **(b)** For a pump operating speed of 1750 rpm, what type of pump (radial-flow, mixed-flow, or axial-flow) would you recommend for this application?



■ **Figure P12.44**

12.45 The axial-flow pump shown in Fig. 12.19 is designed to move 18,900 L/min of water over a head rise of 1.5 m of water. Estimate the motor power requirement and the $U_2V_{\theta2}$ needed to achieve this flowrate on a continuous basis. Comment on any cautions associated with where the pump is placed vertically in the pipe.

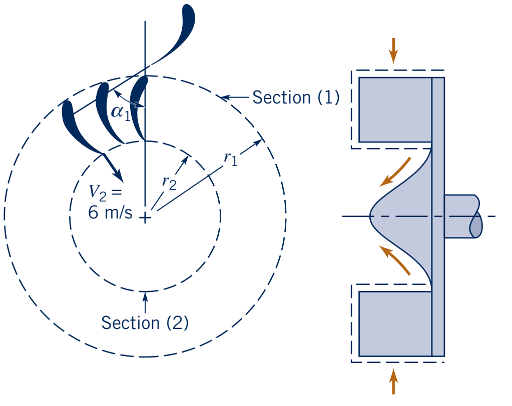
Section 12.7 Fans

12.46 (See Fluids in the News Article titled "Hi-tech Ceiling Fans," Section 12.7.) Explain why reversing the direction of rotation of a ceiling fan results in airflow in the opposite direction.

12.47 For the fan of both Examples 5.19 and 5.28 discuss what fluid flow properties you would need to measure to estimate fan efficiency.

Section 12.8 Turbines (also see Sec. 12.3)

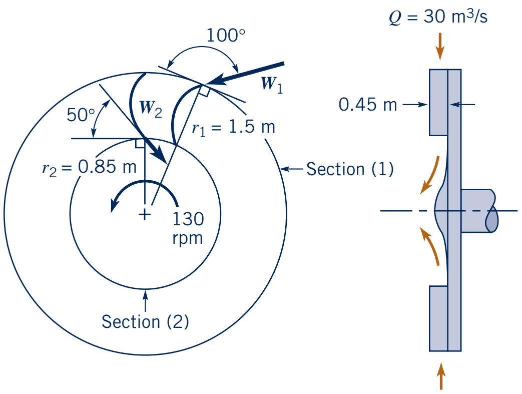
12.48 An inward-flow radial turbine (see Fig. P12.48) involves a nozzle angle, α_1, of 60° and an inlet rotor tip speed, U_1, of 3 m/s.



■ **Figure P12.48**

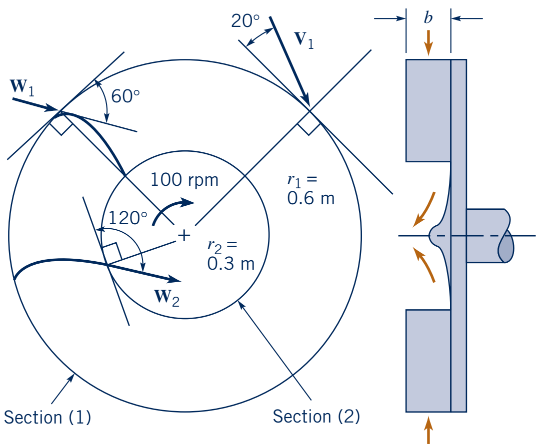
The ratio of rotor inlet to outlet diameters is 2.0. The absolute velocity leaving the rotor at section (2) is radial with a magnitude of 6 m/s. Determine the energy transfer per unit mass of fluid flowing through this turbine if the fluid is **(a)** air, **(b)** water.

12.49 A simplified sketch of a hydraulic turbine runner is shown in Fig. P12.49. Relative to the rotating runner, water enters at section (1) (cylindrical cross section area A_1 at $r_1 = 1.5$ m) at an angle of 100° from the tangential direction and leaves at section (2) (cylindrical cross section area A_2 at $r_2 = 0.85$ m) at an angle of 50° from the tangential direction. The blade height at sections (1) and (2) is 0.45 m, and the volume flowrate through the turbine is 30 m^3/s. The runner speed is 130 rpm in the direction shown. Determine the shaft power developed. Is the shaft power greater or less than the power lost by the fluid? Explain.



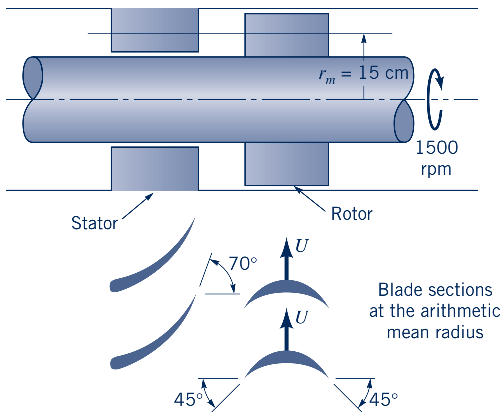
■ **Figure P12.49**

12.50 A water turbine wheel rotates at the rate of 100 rpm in the direction shown in Fig. P12.50. The inner radius, r_2, of the blade row is 0.3 m, and the outer radius, r_1, is 0.6 m. The absolute velocity vector at the turbine rotor entrance makes an angle of 20° with the tangential direction. The inlet blade angle is 60° relative to the tangential direction. The blade outlet angle is 120°. The flowrate is 0.3 m^3/s. For the flow tangent to the rotor blade surface at inlet and outlet, determine an appropriate constant blade height, b, and the corresponding power available at the rotor shaft. Is the shaft power greater or less than the power lost by the fluid? Explain.



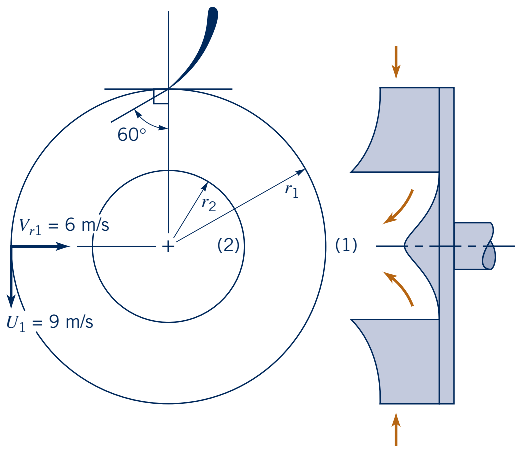
■ **Figure P12.50**

12.51 A sketch of the arithmetic mean radius blade sections of an axial-flow water turbine stage is shown in Fig. P12.51. The rotor speed is 1500 rpm. **(a)** Sketch and label velocity triangles for the flow entering and leaving the rotor row. Use **V** for absolute velocity, **W** for relative velocity, and **U** for blade velocity. Assume flow enters and leaves each blade row at the blade angles shown. **(b)** Calculate the work per unit mass delivered at the shaft.



■ **Figure P12.51**

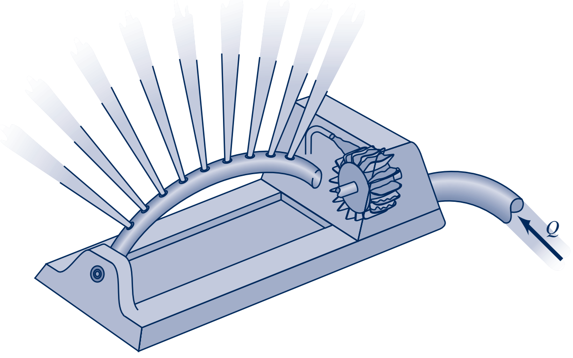
12.52 An inward flow radial turbine (see Fig. P12.52) involves a nozzle angle, α_1, of 60° and an inlet rotor tip speed, U_1, of 9 m/s. The ratio of rotor inlet to outlet diameters is 2.0. The radial component of velocity remains constant at 6 m/s through the rotor, and the flow leaving the rotor at section (2) is without angular momentum. If the flowing fluid is water and the stagnation pressure drop across the rotor is 110 kPa, determine the loss of available energy across the rotor and the efficiency involved.



■ **Figure P12.52**

12.53 Consider the Pelton wheel turbine illustrated in Figs. 12.24, 12.25, 12.26, and 12.27. This kind of turbine is used to drive the oscillating sprinkler shown in **Video V12.4.** Explain how this kind of sprinkler is started, and subsequently operated at constant oscillating speed. What is the physical significance of the zero torque condition with the Pelton wheel rotating?

12.54 A small Pelton wheel is used to power an oscillating lawn sprinkler as shown in **Video V12.4** and Fig. P12.54. The arithmetic mean radius of the turbine is 2.5 cm and the exit angle of the blade is 135° relative to the blade motion. Water is supplied through a single 0.5 cm diameter nozzle at a speed of 15 m/s. Determine the flowrate, the maximum torque developed, and the maximum power developed by this turbine.

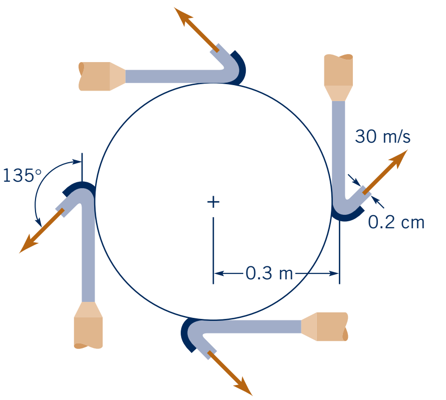


■ **Figure P12.54**

12.55 A Pelton wheel turbine is illustrated in Fig. P12.55. The radius to the line of action of the tangential reaction force on each vane is 0.3 m. Each vane deflects fluid by an angle of 135° as indicated. Assume all of the flow occurs in a horizontal plane. Each of the four jets shown strikes a vane with a velocity of 30 m/s and a stream diameter of 2.5 cm. The magnitude of velocity of the jet remains constant along the vane surface.

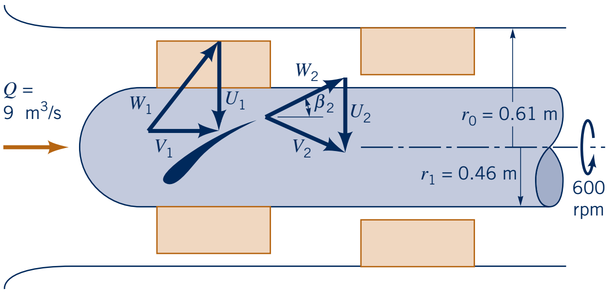
(a) How much torque is required to hold the wheel stationary?

(b) How fast will the wheel rotate if shaft torque is negligible, and what practical situation is simulated by this condition?



■ **Figure P12.55**

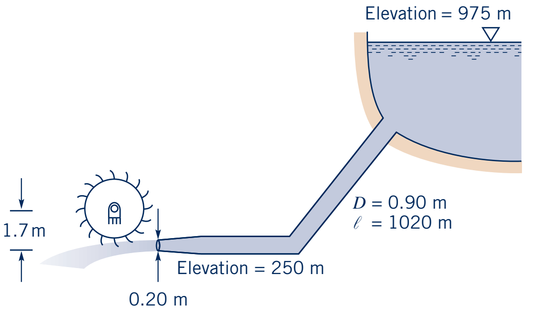
12.56 The single-stage, axial-flow turbomachine shown in Fig. P12.56 involves water flow at a volumetric flowrate of 9 m^3/s. The rotor revolves at 600 rpm. The inner and outer radii of the annular flow path through the stage are 0.46 and 0.61 m, and $\beta_2 =$ 60°. The flow entering the rotor row and leaving the stator row is axial when viewed from the stationary casing. Is this device a turbine or a pump? Estimate the amount of power transferred to or from the fluid.



■ **Figure P12.56**

12.57 For an air turbine of a dentist's drill like the one shown in Fig. E12.8 and **Video V12.5,** calculate the average blade speed associated with a rotational speed of 350,000 rpm. Estimate the air pressure needed to run this turbine.

12.58 Water for a Pelton wheel turbine flows from the headwater and through the penstock as shown in Fig. P12.58. The effective friction factor for the penstock, control valves, and the like is 0.032, and the diameter of the jet is 0.20 m. Determine the maximum power output.



■ **Figure P12.58**

12.59 A 3 m diameter Pelton wheel operates at 500 rpm with a total head just upstream of the nozzle of 1624 m. Estimate the diameter of the nozzle of the single-nozzle wheel if it develops 2×10^7 W.

12.60 A Pelton wheel has a diameter of 2 m and develops 500 kW when rotating 180 rpm. What is the average force of the water against the blades? If the turbine is operating at maximum efficiency, determine the speed of the water jet from the nozzle and the mass flowrate.

12.61 Water to run a Pelton wheel is supplied by a penstock of length ℓ and diameter D with a friction factor f. If the only losses associated with the flow in the penstock are due to pipe friction, show that the maximum power output of the turbine occurs when the nozzle diameter, D_1, is given by $D_1 = D/(2f\,\ell/D)^{1/4}$.

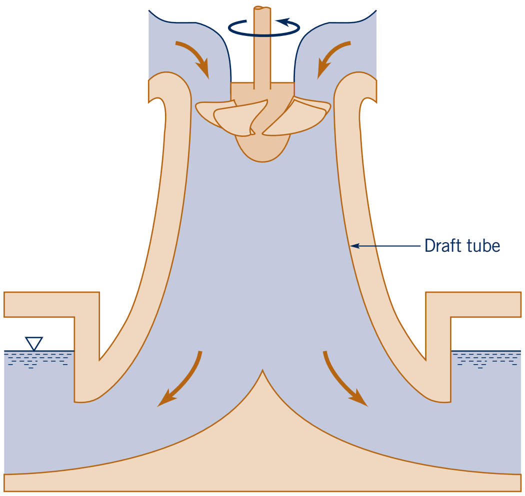
12.62 A Pelton wheel is supplied with water from a lake at an elevation H above the turbine. The penstock that supplies the water to the wheel is of length ℓ, diameter D, and friction factor f. Minor losses are negligible. Show that the power developed by the turbine is maximum when the velocity head at the nozzle exit is $2H/3$. *Note*: The result of Problem 12.61 may be of use.

12.63 If there is negligible friction along the blades of a Pelton wheel, the relative speed remains constant as the fluid flows across the blades, and the maximum power output occurs when the blade speed is one-half the jet speed (see Eq.12.52). Consider the case where friction is not negligible and the relative speed leaving the blade is some fraction, c, of the relative speed entering the blade. That is, $W_2 = cW_1$. Show that Eq. 12.52 is valid for this case also.

12.64 A 1 m diameter Pelton wheel rotates at 300 rpm. Which of the following heads (in meters) would be best suited for this turbine: **(a)** 2, **(b)** 5, **(c)** 40, **(d)** 70, or **(e)** 140? Explain.

12.65 A hydraulic turbine operating at 180 rpm with a head of 30 m develops 1.5×10^7 W. Estimate the power if the same turbine were to operate under a head of 15 m.

12.66 Draft tubes as shown in Fig. P12.66 are often installed at the exit of Kaplan and Francis turbines. Explain why such draft tubes are advantageous.



■ **Figure P12.66**

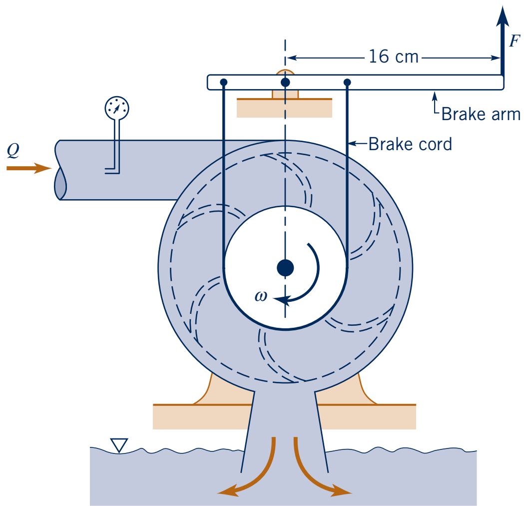
12.67 Turbines are to be designed to develop 22 MW while operating under a head of 21 m and an angular velocity of 60 rpm. What type of turbine is best suited for this purpose? Estimate the flowrate needed.

12.68 Water at 2757 kPa is available to operate a turbine at 1750 rpm. What type of turbine would you suggest to use if the turbine should have an output of approximately 149 kW?

12.69 It is desired to produce 37 MW with a head of 15 m and an angular velocity of 100 rpm. How many turbines would be needed if the specific speed is to be **(a)** 50, **(b)** 100?

12.70 Show how you would estimate the relationship between feature size and power production for a wind turbine like the one shown in **Video V12.1.**

12.71 Test data for the small Francis turbine shown in Fig. P12.71 is given in the following table. The test was run at a constant 9.9 m head just upstream of the turbine. The Prony brake on the turbine output shaft was adjusted to give various angular velocities, and the force on the brake arm, *F*, was recorded. Use the given data to plot curves of torque as a function of angular velocity and turbine efficiency as a function of angular velocity.

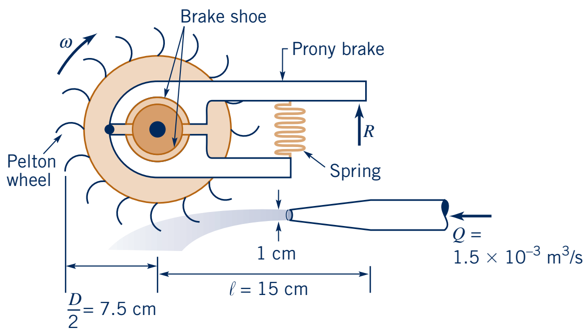


ω (rpm)	Q (10^{-3} m^3/s)	F (N)
0	3.6	12
1000	3.6	11
1500	3.6	10
1870	3.5	8.5
2170	3.3	6.6
2350	2.7	4.0
2580	2.2	1.5
2710	2.0	0.4

■ **Figure P12.71**

†12.72 It is possible to generate power by using the water from your garden hose to drive a small Pelton wheel turbine (see **Video V12.4**). Provide a preliminary design of such a turbine and estimate the power output expected. List all assumptions and show calculations.

12.73 The device shown in Fig. P12.73 is used to investigate the power produced by a Pelton wheel turbine. Water supplied at a constant flowrate issues from a nozzle and strikes the turbine buckets as indicated. The angular velocity, ω, of the turbine wheel is varied by adjusting the tension on the Prony brake spring, thereby varying the torque, T_{shaft}, applied to the output shaft. This torque can be determined from the measured force, R, needed to keep the brake arm stationary as $T_{shaft} = F\ell$, where ℓ is the moment arm of the brake force.



■ **Figure P12.73**

Experimentally determined values of ω and R are shown in the following table. Use these results to plot a graph of torque as a function of the angular velocity. On another graph plot the power output, $W_{\text{shaft}} = T_{\text{shaft}}\,\omega$, as a function of the angular velocity. On each of these graphs plot the theoretical curves for this turbine, assuming 100% efficiency.

Compare the experimental and theoretical results and discuss some possible reasons for any differences between them.

ω (rpm)	R (N)
0	11
360	8.5
450	8.2
600	7.5
700	7.0
940	5.2
1120	4.0
1480	0.7

Section 12.9 Compressible Flow Turbomachines

12.74 Obtain photographs/images of a variety of turbo-compressor rotors and categorize them as axial-flow or radial-flow compressors. Explain briefly how they are used. Note any unusual features.

12.75 Obtain photographs/images of a variety of compressible flow turbines and categorize them as axial-flow or radial-flow turbines. Explain briefly how they are used. Note any unusual features.

■ Lifelong Learning Problems

12.1LL What do you think are the major unresolved fluid dynamics problems associated with gas turbine engine compressors? For gas turbine engine high-pressure and low-pressure turbines? For gas turbine engine fans?

12.2LL Outline the steps associated with the preliminary design of a turbomachine rotor.

12.3LL What are current efficiencies achieved by the following categories of turbomachines? **(a)** Wind turbines; **(b)** hydraulic turbines; **(c)** power plant steam turbines; **(d)** aircraft gas turbine engines; **(e)** natural gas pipeline compressors; **(f)** home vacuum cleaner blowers; **(g)** laptop computer cooling fan; **(h)** irrigation pumps; **(i)** dentist drill air turbines. What is being done to improve these devices?

12.4LL (See Fluids in the News Article titled "Cavitation Damage in Hydraulic Turbines," Section 12.8.2.) How are cavitation and, more importantly, the damage it can cause detected in hydraulic turbines? How can this damage be minimized?